Scottsdale Community College

# College Mathematics Preparation

# Student Workbook

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## ABOUT THIS WORKBOOK

This workbook was created to support mathematics students enrolled in MAT103: College Mathematics Prep at Scottsdale Community College in Scottsdale, Arizona. Any individual may download and utilize a digital copy of this workbook for free. The Creative Commons licensing of this text allows others to freely use, modify, or remix any of the information presented here as long as proper attribution is made. Contact Donna Slaughter at donna.slaughter@scottsdalecc.edu if you are an instructor that wishes to teach with these materials or adapt and edit them for teaching.

As you work through the text, you may find errors. If you do, please contact Donna Slaughter at the email address above and provide specific information about the error and where it is located. The information presented in the text is a work in progress and will evolve with the input of students and other users. We look forward to your feedback!

## WORKBOOK & SUPPORTING COMPONENTS

This workbook is designed to lead students through a series of topics that will prepare students to take College Mathematics. There are six lessons that include the following components:

Lesson Sections with Video Examples and You Try Problems

- Lessons are introduced with practical situations.
- The sections within a lesson provide the lesson instruction.
- **Video Example** problems are to be completed by watching video links and taking notes/writing down the problem as written by the instructor.
- **You Try** problems reinforce lesson concepts and should be worked in the order they appear. Answers can be reviewed in Appendix A.

Practice & Review Problems

- These problems can be found at the end of each lesson. If you are working through this material on your own, the recommendation is to work all those problems. If you are using this material as part of a formal class, your instructor will provide guidance on which problems to complete. Your instructor will also provide information on accessing answers/solutions for these problems.

Online Homework Assessment System

- If you are using these materials as part of a formal class and your class utilizes an online homework/assessment system, your instructor will provide information as to how to access and use that system in conjunction with this workbook.

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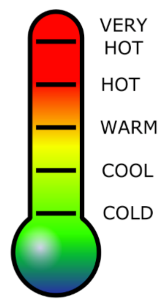
# LESSON 1 – WHOLE NUMBERS & INTEGERS

*On August 10, 2018 Missoula, Montana reported a high temperature of 104 degrees Fahrenheit. In that same year on February 20th, the lowest temperature was recorded at -1 degrees Fahrenheit. How many degrees Fahrenheit were there between the high and low temperature in Missoula, Montana in 2018? *[Answer: 105 degrees]*




In order to answer this question, you will need to have an understanding of both whole numbers and integers. You will be working with both types of numbers as you move through the sections in Lesson 1.

Section 1.1: Types of Whole Numbers
Section 1.2: Whole Numbers and Exponents
Section 1.3: Whole Numbers and Roots
Section 1.4: Integers
Section 1.5: Mathematical Operations with Integers
Section 1.6: Integers and Exponents
Section 1.7: Order of Operations with Integers
Section 1.8: Applications with Whole Numbers and Integers

**Temperature data source: https://www.ncdc.noaa.gov/*

*Image source (public domain images):* https://freesvg.org/thermometer-efl

## Section 1.1: Types of Whole Numbers

*Whole numbers* are numbers we use often in everyday life. We can write a representative set of whole numbers as $\{0, 1, 2, 3, 4, 5\ldots\}$. All whole numbers larger than 1 can be categorized either as prime numbers or composite numbers as indicated below.

- *Prime numbers* are numbers divisible only by themselves and 1. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17.
- *Composite numbers* are divisible by numbers other than themselves and 1 and can be written as a product of their prime factors.

**Video Example:** Identify each of the following numbers as prime or composite. If the number is composite, write it as a product of its prime factors.

a. 32 b. 78 c. 145 d. 31

**You Try:**
Complete the multiplication table below. Use your calculator as needed. A few blocks are completed to get you started. Check your answers in Appendix A.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | | | |
| 1 | | 1 | | | | | | | | | |
| 2 | | | 4 | | | | | | | | |
| 3 | | | | | | | | | | | |
| 4 | | | | | | 20 | | | | | |
| 5 | | | | | | | | | | | |
| 6 | | | | | | | | | | | |
| 7 | | | | | | | | | | | |
| 8 | | | | 24 | | | | | | | |
| 9 | | | | | | | | | | | 90 |
| 10 | | | | | | | | | | | |

## Section 1.2: Whole Numbers and Exponents

*Exponents* are also called *powers* and indicate repeated multiplication.

Read $3^4$ as "three to the fourth power".

Compute as follows: $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 3 \cdot 3 = 27 \cdot 3 = 81$

On your calculator, you can compute exponents a couple of ways as follows:

a) If you are raising a number to the second power (also called "squaring"), look for an $x^2$ key on your calculator. For example, $4^2$, type 4 then $x^2$ ENTER and you should get 16.
b) If you are raising a number to a power other than 2, look for a carrot key (^). For example, $4^5$, type 4^5 ENTER and you should get 1024. Note that you can also use the (^) key even when raising to the 2nd power.

**Video Example:** Write each of the following in expanded form then compute using your calculator as needed.

a. $4^3$ b. $3^5$ c. $12^3$ d. $6^4$

It's good to get used to some of the more common exponent computations. See if you notice any patterns that emerge as you work through the table below.

**Video Example:** Complete the table below using your calculator as needed.

| Exponent | $1^x$ | $2^x$ | $3^x$ | $4^x$ | $5^x$ | $6^x$ | $7^x$ | $8^x$ | $9^x$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | |
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | | | | | | | | | |

From the table above, the following patterns emerged:

- The number 1, raised to any power, is always equal to 1 (column 1).
- Any number raised to the 0 power equals 1 (row 1).
- Any number raised to the 1st power always equals itself (row 2).

**You Try:** Compute each of the following using your calculator as needed and/or reading from the table above. Check your answers in Appendix A.

a. $5^0$ b. $12^2$ c. $7^7$ d. $8^1$

## Section 1.3: Whole Numbers and Roots

*Roots* undo repeated multiplication (i.e. exponents). Some specific roots are listed below.

- The *square root* of a number is that number which, when multiplied times itself, gives the original number. On your calculator, look for $\sqrt{\ }$ to compute square roots.

$$\sqrt{16} = 4 \text{ because } 4 \cdot 4 = 16$$

- The *cube root* of a number is that number which, when multiplied times itself three times, gives the original number. Note the placement of the "3" to indicate cube root.

$$\sqrt[3]{125} = 5 \text{ because } 5 \cdot 5 \cdot 5 = 125$$

On your calculator, you can compute roots a couple of ways as follows:

a. If you are finding a square root, look for a key on your calculator that looks like $\sqrt{\ }$. For example, $\sqrt{16}$ on the TI83/84, press the 2nd function key, then $x^2$ key then 16 ENTER and you should get 4.

b. If you are finding a root other than a 2nd (square) root, on the TI 83/84 look in the Math menu at option 5: $\sqrt[x]{\ }$ . For example, $\sqrt[3]{125}$, type 3 then Math menu option 5: $\sqrt[x]{\ }$ ENTER then 125 ENTER and you should get 5.

**Video Example:** Find each of the roots below. Use your calculator as needed. The table on the previous page may help you with some of these.

a. $\sqrt{64}$ b. $\sqrt[3]{216}$ c. $\sqrt[4]{4096}$ d. $\sqrt{81}$

**You Try:** Find each of the roots below. Use your calculator as needed. The table on the previous page may help you with some of these. Check your answers in Appendix A.

a. $\sqrt{225}$ b. $\sqrt[3]{343}$ c. $\sqrt[4]{1296}$ d. $\sqrt{144}$

## Section 1.4: Integers

If Fred has $\$200$ in his checking account and he writes a check for $\$250$, how could we represent his account balance? We could say that:

$$\text{Fred's Balance} = \$200 - \$250$$

But how can we subtract a number that is larger from one that is smaller? Fred's account is at a deficit status of $-\$50$. We would say his account balance is $-\$50$.

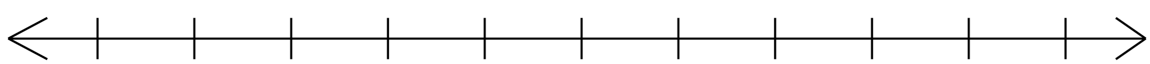
$$\text{Fred's Balance} = \$200 - \$250 = -\$50$$

In order to deal with situations such as this one, whole numbers are not enough. We need to also understand negative whole numbers. Together, whole numbers and their negatives comprise a larger system of numbers called *Integers*. We can write a representative set of integers using the following notation (note that 0 does not have a negative but all the other whole numbers do):

$$\{\ldots -5,\ -4,\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ 4,\ 5\ldots\}$$

When we compare integers, we do so the same way we compare whole numbers. Numbers further to the right on a number line are greater than numbers on the left.

**Video Example:** Label the integers from $-5$ to $5$ on the graph below. Use the graph to help you compare the indicated integers or integer expressions and insert the correct symbol (>, <, =).



a. $-3$____$-5$ b. $\sqrt{4}$____$-2$ c. $-3$____$-1$ d. $(-2)^2$____$(2)^2$ e. $-1^3$____$(-1)^3$

Similar to the bank example that we started with, the examples below illustrate the power of integers to describe real-world situations.

**Video Example:** Provide a signed numerical quantity that accurately represents each of the following situations.

a. Tom gambled in Vegas and lost $52.50. ___________________

b. Larry added 25 songs to his playlist. ___________________

c. The airplane descended 500 feet to avoid turbulence. ___________________

**Video Example:** Camden, SC had a record low temperature of -19°F on Jan 21, 1985, and Monahans, TX had a record low temperature of -23°F on Feb 8, 1933. Which of the two temperatures was the lowest and by how many degrees?
(*Data Source Wikipedia: http://en.wikipedia.org/wiki/U.S._state_temperature_extremes*)

**You Try:** Work each of the following as indicated in the directions. Check your answers in Appendix A.

a. Determine the signed number that best describes each of the statements below:

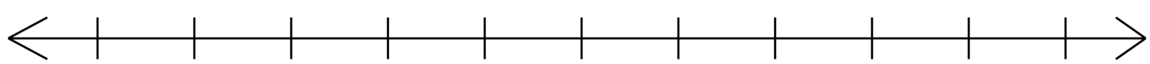
______ A balloon dropped 59 feet.

______ Lori added 324 songs to her iTunes playlist.

______ You owe your friend $235.34.

______Your checking account has a balance of $235.34.

b. Label the integers from -5 to 5 on the graph below. Use the graph to help you compare the indicated integers or integer expressions and insert the correct symbol (>, <, =). Simplify the expression if possible.



a. $-1$____$-2$ b. $-1^2$ ____ $-1$ c. $\sqrt{9}$ ___3 d. $-5$____$-4$

c. Two rides at the amusement park claim to be the greatest thrill. One ride, the Chaotic Coaster drops 100 feet from its highest point. The other ride, the Fearless Flyer, drops 89 feet from its highest point. Which ride drops the most and by how much?

## Section 1.5: Mathematical Operations with Integers

Addition, subtraction, multiplication, and division are called *mathematical operations*. We use mathematical operations to create *mathematical expressions*. In the examples below, you will learn how to simplify mathematical expressions involving integers.

**Video Example:** Apply the indicated operation to simplify the expressions below using your calculator as needed.

a. $6-2$ b. $-4+6$ c. $-3-4$ d. $10-12$

The chart below represents all possible combinations of signs when working with integer multiplication and division. Use the rules to "resolve" the signs when simplifying expressions. The mathematical concepts and procedures behind these rules are much more involved than the information presented here. This method of sign combining is presented as a quick way to simplify more complex expressions.

$$(-)(-) = + \quad (-)(+) = - \quad (+)(-) = - \quad (+)(+) = +$$

$$\frac{(-)}{(-)} = + \quad \frac{(-)}{(+)} = - \quad \frac{(+)}{(-)} = - \quad \frac{(+)}{(+)} = +$$

**Video Example:** Apply the indicated operation to simplify the expressions below using your calculator as needed.

a. $-5+(-2)$ b. $4-(-3)$ c. $2-(+4)$ d. $-8+(+2)$

**Video Example:** Apply the indicated operation(s) to simplify the expressions below using your calculator as needed.

a. $(-8)(-1)$ b. $2(-3)$ c. $(0)(-3)(2)$ d. $(-4)(3)(-1)$

**Video Example:** Apply the indicated operation to simplify the expressions below using your calculator as needed.

a. $8 \div (-4)$ b. $-12 \div (-3)$ c. $\frac{-36}{9}$ d. $\frac{0}{-7}$

In mathematics, the operation of multiplication can be communicated a number of different ways. Each of the notations below communicates the product of 3 times 5.

$$3\text{x}5 = (3)(5) = 3(5) = 3 \bullet 5$$

Similarly, the operation of division can be communicated a number of different ways. Each of the notations below communicates the quotient of 3 divided by 5.

$$3 \div 5 = \frac{3}{5} = {}^{3}\!/_{5} = 3/5$$

**You Try:** Apply the indicated operation to simplify the expressions below using your calculator as needed. Check your answers in Appendix A.

a. $4 - 9$

b. $6 + (-6)$

c. $-5 - 8$

d. $22 - 36$

e. $-3 - (+1)$

f. $7 + (-2)$

g. $-2 - (+3)$

h. $9 - (-12)$

i. $(0)(-5)$

j. $(-1)(-4)$

k. $(-2)(-2)$

l. $(2)(-6)(-3)$

m. $16 \div (-1)$

n. $0 \div (-4)$

o. $\frac{12}{-3}$

p. $\frac{-15}{5}$

## Section 1.6: Integers and Exponents

Raising a negative number to a power requires you to pay very close attention to details like parentheses and placement of the negative sign or signs.

**Video Example:** Compute each of the following. Check using your calculator.

a. $(-1)^2$ b. $-1^2$ c. $(-1)^3$ d. $-1^3$

e. $-(-1)^2$ f. $-(-1)^3$ g. $(-1)^{16}$ h. $(-1)^{17}$

In the example below, we revisit a table similar to one that we computed in Section 1.2.

**Video Example:** Compute each of the following using your calculator as needed.

| Exponent | $(-1)^x$ | $(-2)^x$ | $(-3)^x$ | $(-4)^x$ | $(-5)^x$ | $(-6)^x$ | $(-7)^x$ | $(-8)^x$ | $(-9)^x$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | |
| 1 | | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | | | | | | | | | |
| 4 | | | | | | | | | |
| 5 | | | | | | | | | |

Some patterns continue from the table we computed in Section 1.2:
- Any number raised to the 0 power equals 1.
- Any number raised to the 1st power always equals itself.

However, there are some new patterns as follows:
- Any negative number raised to an even power (2, 4 and above) gives a positive number result.
- Any negative number raised to an odd power (1, 3, 5 and above) gives a negative number result.

**You Try:** Compute each of the following using your calculator as needed. Check your answers in Appendix A. Use the table above to help you.

a. $(-2)^2$ b. $-2^2$ c. $(-2)^3$ d. $-2^3$

e. $-(-2)^2$ f. $-(-2)^3$ g. $(-2)^4$ h. $(-2)^5$

## Section 1.7: Order of Operations with Integers

When presented with more than one mathematical operation in an expression, we need to know which one to address first. The letters PEMDAS will help us remember the correct order to follow.

1. P = Simplify items inside Parentheses ( ), brackets [ ] or other grouping symbols first.
2. E = Simplify items that are raised to powers (Exponents)
3. MD = Perform Multiplication and Division in order of appearance from **Left to Right**
4. AS = Perform Addition and Subtraction in order of appearance from **Left to Right**

**Video Example:** Use PEMDAS to simplify the expressions below using your calculator as needed.

a. $8+5\bullet 2$

b. $-20-20+(-10)$

c. $8-(-2)+4+(-3)-2$

d. $7-(-2)+(-1)-5$

**Video Example:** Use PEMDAS to simplify the expressions below using your calculator as needed.

a. $-6\div(-1)^2+2\bullet(-3)$

b. $24\div(2-(-4))$

c. $20-(8-2)\div 3\bullet(-4)$

d. $10\bullet 3^2+(10-4)\div 2$

**You Try:** Use PEMDAS to simplify the expressions below using your calculator as needed. Check your answers in Appendix A.

a. $-6+4\bullet(-2)$

b. $-36\div(4-(-2))$

c. $5-(10-5)\div 5\bullet(-2)$

d. $5\bullet 2^2+(-6-6)\div 3$

## Section 1.8: Applications with Whole Numbers & Integers

Applications (a.k.a. word problems) ask you to use math to solve real-world problems. To approach these problems effectively, the following process will help.
Check the directions for each problem you work to see which parts you need to present as part of your final solution.

Step 1: [Given and Goal Step] Read the problem carefully underlining information that seems important. Writing this information down in a concise form can be useful as well. Diagrams can sometimes help make sense of given information. Also, as part of this step, identify what is you are asked to find. This is your goal.

Step 2: [Computation Step] Begin to work with the information from step 1 and try to formulate a process that will work to achieve the goal. Write down any equations or expressions that will help. Try to compute a result.

Step 3: [Check for Reasonableness] Once you have a result of some kind, ask yourself if the answer seems reasonable for the situation. Does your result fit the context of the problem and the goal?

Step 4: [Formal Statement of your Result] Write out a COMPLETE SENTENCE that includes your final result from step 2 and addresses the goal from step 1. Include units if they are present in your problem.

**Video Example:** Ryan has an outstanding balance of $2,312 on his credit card.
If he makes charges totaling $324 and makes a payment of $425, what is his new balance after his payment?

**Video Example:** Amber is responsible for providing breakfast to a large group of campers. Hashed browns are on her list of supplies to purchase and she needs to buy enough to serve 100 people. The hashed browns are sold in packs of 8 boxes and each box in the pack will serve 4 people. How many packs of hashed browns should she purchase in order to serve 100 people? Will she have any hashed browns left over?

**You Try:**

a. You join a local swim club that meets 3 times per week to swim laps. The initial enrollment fee is $105 and the group membership is $44 a month. What are your dues for the first year of membership?

b. On the first five rolls of your Farkle game in Facebook, you earned -400, 1250, 0, 0, -500 points. What is your total after the 5 rolls?

c. Alice bought dog food for an animal rescue shelter. She bought 6 bags that weighed 25-pounds each and 9 bags that weighted 10-pounds each. How may pounds of dog food did she buy?

## Lesson 1 - Practice & Review

1. Simplify each of the following using your calculator as needed.

a. $14+23$ b. $14-23$ c. $(-14)+23$ d. $14-(-23)$

e. $(-4)^2$ f. $-4^2$ g. $(-4)^3$ h. $-4^3$

i. $-(-4)^2$ j. $-(-4)^3$ k. $(-4)^4$ l. $(-4)^5$

m. $\sqrt{100}$ n. $\sqrt[3]{512}$ o. $\sqrt[4]{256}$ p. $\sqrt{169}$

2. Simplify each of the following using your calculator as needed.

a. $5-8$ b. $4+(-4)$ c. $-3-5$ d. $20-16$

e. $-2-(+2)$ f. $6+(-2)$ g. $-3-(+2)$ h. $7-(-11)$

i. $(0)(-3)$ j. $(-1)(-5)$ k. $(-3)(-3)$ l. $(3)(-4)(-5)$

m. $-14\div(-1)$ n. $0\div(-3)$ o. $\dfrac{16}{-4}$ p. $\dfrac{-25}{5}$

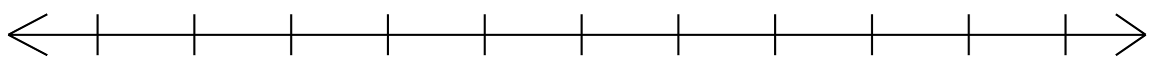
3. Determine the signed number that best describes each of the statements below:

______ a. James owes his friend $25.13.

______ b. Sirina let 3 pounds of air pressure out of her tire.

______ c. The temperature rose 15 degrees between 8 am and 4 pm.

4. Label the integers from -5 to 5 on the graph below. Use the graph to help you compare the indicated integers or integer expressions and insert the correct symbol (>, <, =).



a. $-4$ _____ $-5$ b. $-1^2$ _____ $1$ c. $\sqrt{16}$ ___ $3$ d. $(-2)^2$ _____ $4$

5. Two friends set goals to lose weight over a 30-day period. Over the 30 days, Anderson lost 5 lbs., gained 4 lbs., lost 3 lbs., gained 2 lbs., lost 3 lbs., and gained 1 lb. Brian gained 2 lbs., lost 3 lbs., gained 1 lb., lost 6 lbs., lost 2 lbs., and gained 1 lb. How much weight total did each friend gain or lose during the 30 days?

6. Use PEMDAS to simplify the expressions below showing steps. Use your calculator as needed to check your work.

a. $4-3\bullet(-1)$

b. $-15\div(3-(-2))$

c. $4+(-6-4)\div5\bullet(-3)$

d. $3\bullet(-2)^2+(-5-5)\div2$

7. Solve each application problem below and write your final answer as a complete sentence.

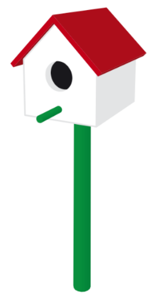
a. Lance bought 3 packs of hamburger buns. Each pack contained 12 buns. How many buns did he buy?

b. An airplane took off and achieved a cruising altitude of 35,000 feet. About halfway through its’ 3-hour flight, the plane descended 2000 feet to avoid turbulence then ascended 3500 feet to a new cruising altitude. What was the new cruising altitude?

c. Lars takes out a $22,400 student loan to pay his expenses while he is in college. After graduation, he will begin making payments of $249 per month for the next 10 years to pay off the loan. How much more will Lars end up paying for the loan than the original value of $22,400?

# LESSON 2 – FRACTIONS

*A woodworker is making a bird house and needs to cut pieces that are $\frac{1}{2}$ ft. long from boards that are 4 feet long. How many pieces can he cut from one board that is 4 feet long? *[Answer: 8 pieces]*



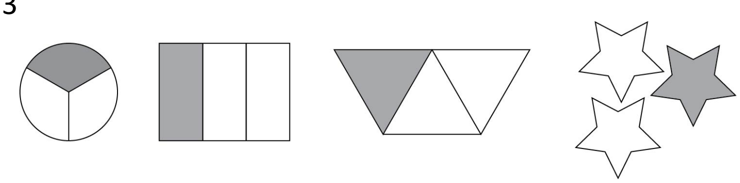
In order to answer this question, you will need to have understand how to divide and work with factions. In this lesson, you will learn what fractions are, how to represent them, how to simplify them, and how to combine them.



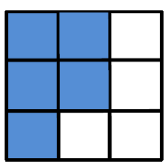
** Image source (public domain images):*
https://freesvg.org/birdhouse-155398

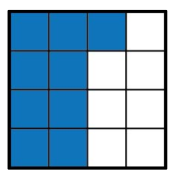
## Section 2.1: Fractions and Mixed Numbers

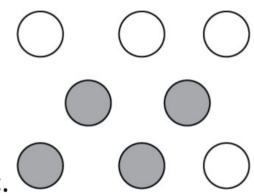
Suppose I buy a candy bar to split with two of my friends. What number could we use to discuss how much of the bar each of us would get? If we have 1 bar and split into 3 equal pieces, then we say that each person gets $\frac{1}{3}$ (one-third) of the bar. The *fraction* $\frac{1}{3}$ is the number that represents one part of a whole that is divided into 3 equal pieces. The fraction $\frac{1}{3}$ can be represented by the shaded part in each of the following diagrams. Notice that in each diagram, the whole is a different shape or set of shapes but the use of the fraction $\frac{1}{3}$ still applies.



**Video Example:** Identify the fraction represented by the shaded part of each shape or set of shapes.

a. 

b. 

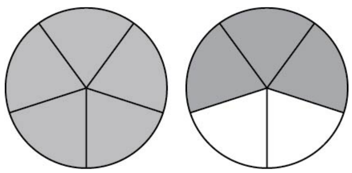
c. 

**You Try:** Draw two different shapes or sets of shapes that represent the fraction $\frac{3}{4}$.

Vocabulary of fractions:

- The top number in a fraction is called the *numerator.*
- The bottom number in a fraction is called the *denominator.*
- If the numerator is smaller than the denominator, we have a *proper fraction.*
- If the numerator is larger than the denominator, we have an *improper fraction.*
- Improper fractions are the only fractions that can be written as *mixed numbers*.

**Video Example:** Identify the fraction represented by the shaded part of the image.



**Video Example:** Express each of the following as an improper fraction using your calculator as needed.

a. $2\frac{1}{4}$ b. $12\frac{1}{3}$ c. $1\frac{1}{5}$ d. 7

**Video Example:** Express each of the following as a mixed number using your calculator as needed.

a. $\frac{12}{5}$ b. $\frac{20}{7}$ c. $\frac{24}{6}$ d. $\frac{3}{1}$

**You Try:**

a. Write steps to convert $3\frac{1}{2}$ to an improper fraction.

b. Express each of the following as an improper fraction using your calculator as needed.

i. $3\frac{1}{3}$ ii. $8\frac{2}{7}$ iii. $4\frac{2}{5}$ iv. $3$

c. Write steps to convert $\frac{57}{11}$ to a mixed number.

d. Express each of the following as a mixed number using your calculator as needed.

i. $\frac{17}{4}$ ii. $\frac{43}{6}$ iii. $\frac{36}{9}$ iv. $\frac{8}{1}$

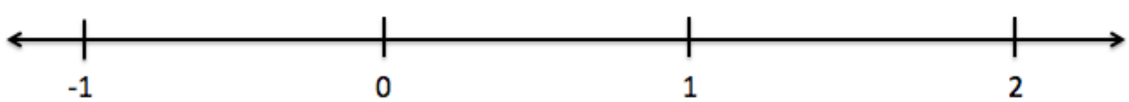
## Section 2.2: Simplifying and Graphing Fractions

Fractions are said to be in *simplest form* if they are completely reduced. To completely reduce a fraction, remove all common factors other than 1 from the numerator and denominator. Leave fraction answers always in simplest form.

**Video Example:** Write the following fractions in simplest form using your calculator as needed.

a. $\frac{4}{16}$ b. $\frac{28}{54}$ c. $\frac{22}{6}$ d. $\frac{-45}{5}$

**Video Example:** Divide the given line into units of length $\frac{1}{6}$, then plot and label the following fractions: $-\frac{3}{6},-\frac{1}{6},\frac{0}{6},\frac{2}{6},\frac{5}{6},\frac{6}{6},\frac{9}{6}$. Provide simplified forms for each fraction if possible.




You may have noticed different placements of negative signs within the examples in this lesson. The following are all equivalent forms of the fraction "negative a over b". However, the form of a fraction that has the negative in front (the last one in the chain below) is considered to be a more "simplified" form and is generally how we leave our fractions.

$$\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b}$$

This is the idea with a numerical example:

$$\frac{-1}{2}=\frac{1}{-2}=-\frac{1}{2}$$

**Video Example:** Compute each of the following using your calculator as needed.

a. $\frac{4}{1}$ b. $\frac{4}{4}$ c. $\frac{0}{4}$ d. $\frac{4}{0}$

What are some general rules we can identify given the results above?

- Any number divided by 1 is itself.
- Any nonzero number divided by itself is 1.
- Zero divided by any nonzero number is zero.
- Any number divided by zero is not defined.

**You Try:**

a. Write the following fractions in simplest form using your calculator as needed.

i. $\frac{-12}{36}$ ii. $\frac{32}{54}$ iii. $\frac{20}{12}$ iv. $\frac{-40}{10}$

b. Compute each of the following using your calculator as needed.

i. $\frac{0}{3}$ ii. $\frac{6}{6}$ iii. $\frac{7}{0}$

## Section 2.3: Applications with Fractions

The problem-solving process we learned in Section 1.8 applies to any application problem you are trying to solve, including those that involve fractions. Use that process to help you in this section.

**Video Example:**

a. There are 14 men and 12 women in Professor Bohart's MAT114 class.
What fraction of the students in the class are women?

b. The local PTA group approved a fall carnival by a vote of 15 to 5. What fraction of the PTA group voted against the bill?

**You Try:**

a. Jorge's family has visited 38 of the 50 states in America. What fraction of the states have they visited?

b. John had 12 marbles in his collection. Three of the marbles were Comet marbles. What fraction of the marbles were Comet marbles? What fraction were NOT Comet marbles?

## Section 2.4: Adding & Subtracting Fractions with Like Denominators

Let's revisit the candy bar example from Section 2.1. Remember that you had one candy bar split between yourself and two friends so each of you received $\frac{1}{3}$ of the bar. Between your two friends, how much of the bar did they have together? They each had $\frac{1}{3}$ of the bar meaning together they had $\frac{2}{3}$ of the bar. What does this look like mathematically?

$$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

Notice that when we add $\frac{1}{3}$ and $\frac{1}{3}$, we end up with $\frac{2}{3}$. Because each fraction has the same denominator, to add them we just add the numerators.

If we add your piece back, how much of the bar would there be between the three of you?

$$\frac{2}{3}+\frac{1}{3}=\frac{3}{3}=1$$

Again, we are working with the same denominator of 3, so $\frac{2}{3}$ combined with $\frac{1}{3}$ gives $\frac{3}{3}$ which simplifies to 1 (i.e. the entire candy bar).

This is a very simple example that illustrates the most critical concept behind combining fractions through addition and subtraction and that is…

TO ADD OR SUBTRACT FRACTIONS, THE DENOMINATORS MUST BE THE SAME.

And remember to always apply the rule of fraction simplification which says that,

FINAL FRACTION RESULTS MUST ALWAYS BE PRESENTED IN SIMPLEST FORM.

**Video Example:** Add the fractions below. Write your final answer in simplest form. Use your calculator to check your work.

a. $\frac{5}{12}+\frac{3}{12}$

b. $\frac{3}{8}+\frac{7}{8}$

c. $-\frac{3}{4}+\frac{11}{4}$

d. $1\frac{1}{5}+\frac{2}{5}$

**Video Example:** Subtract the fractions below. Write your final answer in simplest form. Use your calculator to check your work.

a. $\frac{5}{8}-\frac{2}{8}$

b. $\frac{11}{12}-\frac{7}{12}$

c. $\frac{3}{4}-\frac{9}{4}$

d. $-\frac{27}{6}+\frac{27}{6}$

**You Try:** Add or subtract as indicated. Write your final answer in simplest form. Use your calculator to check your work.

a. $\frac{2}{3}+\frac{5}{3}$

b. $\frac{2}{5}+\frac{8}{5}$

c. $2\frac{3}{4}+1\frac{2}{4}$

d. $-\frac{1}{5}+\frac{2}{5}$

e. $\frac{8}{13}-\frac{2}{13}$

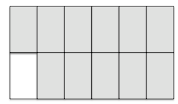
f. $\frac{11}{12}-\frac{14}{12}$

g. $\frac{9}{4}-\frac{3}{4}$

h. $\frac{7}{6}-\frac{3}{6}$

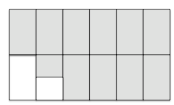
## Section 2.5: Adding and Subtracting Fractions with Unlike Denominators

Josh made a pan of chocolate brownies to share with friends at a party. He cut the pan of brownies into 12 equal pieces. While waiting for his friends to arrive, he could not resist, so he ate one brownie. What fraction of the brownies are gone?

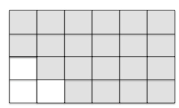


Josh ate $\frac{1}{12}$ of the brownies.

His friends were late and those brownies were good!!!!!! But he did not want to eat them all so he just took half of a second brownie and ate that. What fraction of the brownies are now missing?



Because the second piece was not the same size as the first, we cannot directly count the number of pieces missing. However, if we cut the brownies into 24 pieces instead of 12 (see below), then we can easily see that 3 of the now 24 pieces are missing.



Josh ate $\frac{3}{24}=\frac{1}{8}$ of the brownies.

How do we show this mathematically? Josh ate $\frac{1}{12}$ of the brownies followed by $\frac{1}{24}$ of the brownies. That gives the sum $\frac{1}{12}+\frac{1}{24}$. We know from the previous lesson that denominators must be the same in order to add fractions. Dividing the brownies into 24 pieces helps us see that $\frac{1}{12}$ is equivalent to $\frac{2}{24}$. We can use this information to create a sum with like denominators as follows: $\frac{2}{24}+\frac{1}{24}=\frac{3}{24}=\frac{1}{8}$. So, Josh ate $\frac{3}{24}=\frac{1}{8}$ of the brownies before his friends arrived.

This example illustrates again the most critical concept behind combining fractions through addition and subtraction and that is…

TO ADD OR SUBTRACT FRACTIONS, THE DENOMINATORS MUST BE THE SAME.

The video examples that follow will explain more clearly how to create equivalent fractions in order to add or subtract fractions with unlike denominators.

**Video Example:** Add as indicated. Write your final answer in simplest form. Use your calculator to check your work. The multiplication table you developed in Section 1.1 may help you with choosing denominators.

a. $\frac{1}{2}+\frac{2}{3}$

b. $\frac{3}{8}+\frac{5}{6}$

c. $-\frac{2}{5}+\frac{7}{4}$

d. $2\frac{1}{3}+\frac{2}{5}$

**Video Example:** Subtract as indicated. Write your final answer in simplest form. Use your calculator to check your work. The multiplication table you developed in Section 1.1 may help you with choosing denominators.

a. $\frac{2}{3}-\frac{1}{2}$

b. $\frac{3}{4}-\frac{2}{5}$

c. $\frac{1}{5}-\frac{5}{4}$

d. $4-\frac{2}{3}$

**You Try:** Add or subtract as indicated. Write your final answer in simplest form. Use your calculator to check your work. The multiplication table you developed in Section 1.1 may help you with choosing denominators.

a. $\frac{4}{5}+\frac{5}{8}$

b. $\frac{2}{7}+\frac{8}{5}$

c. $1\frac{2}{9}+\frac{4}{5}$

d. $-\frac{4}{9}+\frac{1}{2}$

e. $\frac{4}{5}-\frac{11}{15}$

f. $\frac{11}{15}-\frac{4}{5}$

g. $2-3\frac{3}{4}$

h. $\frac{7}{6}-\frac{2}{5}$

## Section 2.6: Applications with Fraction Addition and Subtraction

The problem-solving process we learned in Section 1.8 applies to any application problem you are trying to solve, including those that involve fractions. Use that process to help you in this section.

**Video Example:**

a. In a bag of 100 M & M's, 30 are brown, 20 are yellow, 20 are red, 10 are green, 10 are orange, and 10 are blue. What fraction of the M & M's are brown or blue?

b. Gabriel flew from Phoenix to Florida recently. His first flight lasted $3\frac{1}{2}$ hours and his second flight lasted $1\frac{1}{4}$ hours. What was his total flying time? Present your result in hours.

**You Try:**

Kayla baked a lemon pie and ate $\frac{1}{4}$ of it. Her sister, Sara, ate $\frac{1}{3}$ of the pie.

a. Together, what fraction of the pie did they eat?

b. What fraction of the pie was left over?

c. Who ate more and by how much?

## Section 2.7: Multiplying Fractions

Let's revisit the example of Josh and the brownies in order to set the stage for fraction multiplication. We ascertained that Josh ate $\frac{1}{8}$ of the brownies when there were 24 brownies in the pan. I can write the following question from this information. "If Josh ate $\frac{1}{8}$ of the brownies in a 24-brownie pan, how many brownies did he eat?" We know from before that he ate 3 brownies (of the 24-brownie per pan size). How do we set that up mathematically? We multiply $\frac{1}{8}$ times 24 as shown below.

$$\frac{1}{8}\bullet 24=\frac{1}{8}\bullet\frac{24}{1}=\frac{24}{8}=3$$

Notice that with fraction multiplication, we do NOT obtain a common denominator prior to multiplying. Here are the steps for fraction multiplication:

1. Convert any whole numbers/mixed numbers to improper fractions.
2. Multiply straight across and write in simplest form.

The video examples will explain this process in greater detail.

**Video Example:** Multiply as indicated. Write your final answer in simplest form. Use your calculator to check your work.

a. $\frac{1}{4}\bullet\frac{3}{2}$

b. $\frac{5}{8}\bullet 4$

c. $2\frac{1}{5}\bullet\frac{7}{4}$

d. $\frac{1}{3}\bullet\frac{2}{3}\bullet\frac{3}{4}$

**You Try:** Multiply as indicated. Write your final answer in simplest form. Use your calculator to check your work.

a. $\frac{2}{5}\bullet\frac{3}{2}$

b. $\frac{7}{11}\bullet 3$

c. $1\frac{2}{3}\bullet 9$

d. $\frac{5}{8}\bullet\frac{2}{3}\bullet\frac{1}{5}$

## Section 2.8: Applications with Fraction Multiplication

The problem-solving process we learned in Section 1.8 applies to any application problem you are trying to solve, including those that involve fractions. Use that process to help you in this section.

**Video Example:**

a. Matt was training for a marathon and had a 20-mile run listed on his training calendar. If he only completed $\frac{3}{4}$ of the training run, how many miles did he complete?

b. Brian earns $10 for every hour he works each week up to 40 hours. Any additional hours are considered overtime and he earns “time and a half” wages. If he worked 56 hours one week, what were his total earnings?

**You Try:**

Darcy’s iTunes library has 3525 songs on it. If $\frac{1}{3}$ of those songs are categorized as Hip Hop/Rap, how many songs are in the Hip Hop/Rap category?

## Section 2.9: Dividing Fractions

Workers at a doggie day-care business watch the dogs in $1\frac{1}{2}$ hour shifts. If the business is open 6 hours each day, how many times do shift changes occur?

Another way to ask the question posed in this problem is, "how many times does $1\frac{1}{2}$ divide into 6?" Let's compute the answer first by adding $1\frac{1}{2}$ to itself until we reach 6.

$$1\frac{1}{2}+1\frac{1}{2}=3$$
$$1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}=4\frac{1}{2}$$
$$1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}=6$$

So, $1\frac{1}{2}$ divides into 6, 4 times meaning there are 4 shift changes every 6 hours. What does the math look like to compute the division directly?

$$6\div1\frac{1}{2}=6\div\frac{3}{2}=6\bullet\frac{2}{3}=\frac{12}{3}=4$$

Notice what happened in the third step. Division by $\frac{3}{2}$ was changed to multiplication by the reciprocal, $\frac{2}{3}$. A simpler example might help you remember why this method works. Let's ask the question, what is $1\div\frac{1}{2}$? In other words, how many $\frac{1}{2}$'s are there in 1? Well, there are 2. How do we get that mathematically? The only way to get from $1\div\frac{1}{2}$ to the correct answer of 2 is to invert the divided fraction and multiply like this:

$$1\div\frac{1}{2}=1\bullet\frac{2}{1}=1\bullet2=2$$

Here are the steps for fraction division:

1. Convert any whole numbers/mixed numbers to improper fractions.
2. Convert division by a fraction to multiplication by the reciprocal of that fraction.
3. Multiply straight across and write final answer in simplest form.

**Video Example:** Divide as indicated. Write your final answer in simplest form. Use your calculator to check your work.

a. $2 \div \frac{1}{4}$

b. $3\frac{1}{2} \div \frac{2}{5}$

c. $\frac{7}{2} \div \frac{3}{4}$

d. $\frac{8}{12} \div 4$

**You Try:** Divide as indicated. Write your final answer in simplest form. Use your calculator to check your work.

a. $\frac{2}{3} \div 10$

b. $\frac{1}{2} \div \frac{2}{7}$

c. $3\frac{1}{4} \div \frac{1}{2}$

d. $4 \div \frac{4}{3}$

## Section 2.10: Applications with Fraction Division

The problem-solving process we learned in Section 1.8 applies to any application problem you are trying to solve, including those that involve fractions. Use that process to help you in this section.

**Video Example:**

a. A recipe for Albondigas Soup that makes 8 servings calls for 3 potatoes, $1\frac{1}{2}$ cups of salsa, and 2 pounds of ground beef as the main ingredients. You only want to make half this amount. How much of each item will you need to include?

b. If you plan to have a pizza party and each guest will eat $\frac{3}{4}$ of a pizza, how many guests can you feed with 12 pizzas?

**You Try:**

Sally was cutting a 100-ft long tree into sections. The sections must be no more than $1\frac{1}{4}$ feet long in order to fit in her fireplace.

a. How many sections of $1\frac{1}{4}$ feet length could she cut?

b. If each section were split into 4 logs, how many logs would she have in total?

## Section 2.11: Order of Operations with Fractions and Integers

When presented with more than one mathematical operation in an expression, we need to know which one to address first. The letters PEMDAS will help us remember the correct order to follow. We used PEMDAS in Lesson 1 with integers and whole numbers. Now we will use it with fractions as well.

5. P = Simplify items inside Parenthesis (), brackets [] or other grouping symbols first.
6. E = Simplify items that are raised to powers (Exponents)
7. MD = Perform Multiplication and Division in order of appearance from **Left to Right**
8. AS = Perform Addition and Subtraction in order of appearance from **Left to Right**

**Video Example:** Use PEMDAS to simplify the expressions below using your calculator as needed.

a. $8+4\bullet\frac{1}{2}-\frac{1}{3}\div 2$

b. $\left(-\frac{1}{4}\right)^2+\frac{1}{2}$

c. $\frac{1}{3}\bullet\frac{-2}{5}\div\frac{5}{6}$

d. $\frac{2}{3}-\left(-\frac{2}{3}\right)+\left(-\frac{1}{4}\right)-4$

**You Try:** Use PEMDAS to simplify the expressions below using your calculator as needed. Check your answers in Appendix A.

a. $7-\frac{3}{4}\bullet\frac{1}{3}+4\div\left(-\frac{1}{2}\right)$

b. $\frac{2}{5}\bullet\frac{3}{4}\div\left(\frac{-4}{7}\right)$

## Lesson 2 - Practice & Review

1. Draw two different shapes or set of shapes that represent the fractions below.

a. $\frac{5}{8}$ b. $\frac{2}{5}$ c. $1\frac{1}{2}$

2. Express each of the following as an improper fraction using your calculator as needed.

a. $1\frac{2}{3}$ b. 5 c. $2\frac{7}{8}$ d. $4\frac{1}{5}$

3. Express each of the following as a mixed number using your calculator as needed.

a. $\frac{21}{3}$ b. $\frac{7}{2}$ c. $\frac{45}{10}$ d. $\frac{6}{2}$

4. Write the following fractions in simplest form using your calculator as needed.

a. $\frac{14}{28}$ b. $-\frac{18}{40}$ c. $\frac{40}{24}$ d. $\frac{-50}{5}$

e. $\frac{12}{3}$ f. $\frac{6}{0}$ g. $\frac{0}{12}$ h. $-\frac{6}{1}$

5. Simplify each of the following expressions by performing the indicated operations. Leave all fractions in simplest form. Use your calculator to help as needed.

a. $\frac{2}{5}+\frac{5}{7}$ b. $\frac{3}{5}-\frac{8}{5}$

c. $3\frac{3}{7}-1\frac{2}{7}$ d. $\frac{11}{17}-\frac{14}{17}$

6. Simplify each of the following expressions by performing the indicated operations. Leave all fractions in simplest form. Use your calculator to help as needed.

a. $\frac{3}{4}-\frac{5}{8}+\frac{1}{2}$

b. $\frac{13}{7}+\frac{8}{5}$

c. $-\frac{2}{3}+\frac{1}{2}$

d. $2-\frac{11}{5}$

e. $\frac{2}{5}\bullet\frac{3}{2}$

f. $\frac{2}{3}\bullet 4$

g. $3\frac{3}{5}\bullet 9$

h. $\frac{1}{2}\div 8$

7. Simplify each of the following expressions by performing the indicated operations. Leave all fractions in simplest form. Use your calculator to help as needed.

a. $6+\frac{2}{5}\bullet\frac{1}{2}+8\div\left(-\frac{1}{4}\right)$

b. $\frac{2}{8}\bullet\frac{5}{3}\bullet\frac{-3}{2}$

c. $\left(-\frac{1}{2}\right)^2-\frac{4}{6}\div\frac{3}{2}\bullet 5\div\left(-\frac{2}{3}\right)$

d. $\frac{1}{4}\bullet\frac{3}{6}\div\frac{-3}{2}$

8. Solve each of the following application problems using the appropriate operations. Be sure to leave your final answer as a complete sentence.

a. Shawn recently went to a one-week summer camp in Oregon to learn wilderness survival skills. The program was broken into themes over the 7 days. Two days were spent on medical training, one day on shelter building, two days on hunting and fishing, one day on foraging for wild edibles, and one day on understanding local predators. What fraction of Shawn's time was spent on medical training and what fraction on foraging for wild edibles?

b. Geri's grandfather gave her a set of 25 old coins from his collection. Ten of these coins were dated with dates before 1945 and the rest were dated between 1945 and 1960. What fraction of the coins were dated with dates before 1945? What fraction were dated between 1945 and 1960?

c. Crazy Cakes Bakery baked a huge batch of chocolate chip cookies to share with local hospitals during the pandemic. They donated $\frac{1}{4}$ of the cookies to Baywood Hospital, $\frac{2}{5}$ of the cookies to La Salle Hospital, and $\frac{3}{10}$ of the cookies to Brookline hospital. If they made 1000 cookies, how many did they donate to each hospital? Were there any cookies left over and if so, how many?

d. Larena works $27\frac{1}{2}$ hours weekly at a local store. If she is scheduled to work five days each week and works the same number of hours each day, how many hours does she work each day?

e. Tina is working at the local fire department and is tasked with cutting a 145-foot firehose into segments that are $5\frac{1}{2}$ feet long. How many segments can she cut? Will there be any left over and if so, how much?

# LESSON 3 – DECIMALS & PERCENTS

*Jeran went to the Ace Grocery Store and purchased items that cost $5.65, $2.35, $4.10, $3.99, and $6.98. The items were taxed at 9%. What was the total bill? *[Answer: $25.15]*

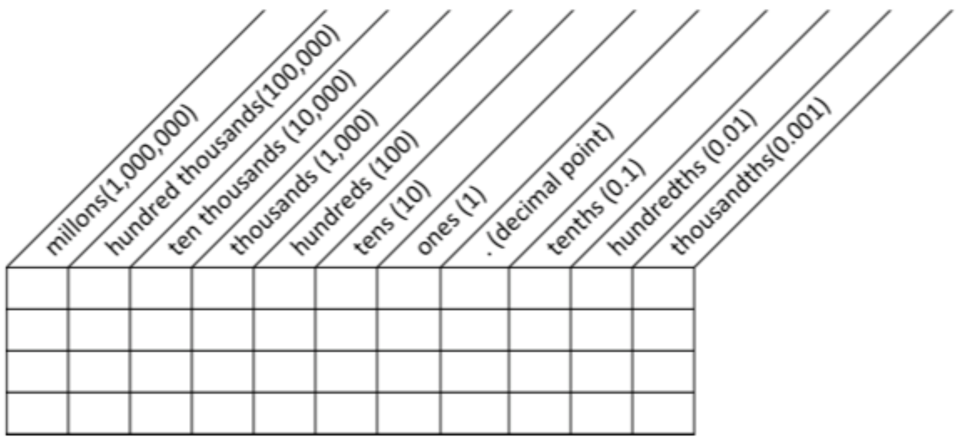

In order to answer this question, you will need to understand how to add decimals and apply the concept of percent. In this lesson, you will learn all about working with decimals as well as the basics of percents and percent equations.



** Image source (public domain images):*
https://freesvg.org/grocery-shopping-bag

## Section 3.1: Decimal Place Value and Rounding

The chart below illustrates place values for numbers from the thousandths place (0.001) all the way to the millions place (1,000,000). Understanding place value allows us to correctly round and estimate numbers and number combinations.



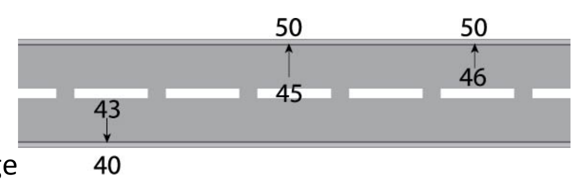

**Video Example:** Write each number in words. What place does the digit "0" occupy in each number?

a. 15.045

b. 320,231.17

c. 256.304

To *round* a number means to approximate that number by replacing it with another number that is "close" in value. Rounding is often used when estimating. When rounding, the analogy of a road may help. See the image below. The numbers 43, 45, and 46 are rounded to the nearest tens place. Note that a number in the middle of the "road" is rounded up.

Image 


**Video Example:**

a. Round 42.345 to the hundredths place.

b. Round 52.3 to the tens place.

c. Round 212.10 to the ones place.

d. Round 42.399 to the tenths place.

ROUNDING PROCESS

1. Round UP when the digit to the right of the place value we want to round to is 5 or greater.
2. Round DOWN when the digit to the right of the place value we want to round to is less than 5.

**You Try:**
Work each of the following. Check your answers in Appendix A.

a. Write the number 12.617 using words then round to the hundredths place. What place does the digit "6" occupy in 12.617?

b. Write the number 3,465.05 using words then round to the tens place. What place does the digit "4" occupy in 3,465.05?

c. Write the number 525.467 using words then round to the hundreds place. What place does the digit "7" occupy in 525.467?

d. Write the number 12,456.99 using words then round to the ones place. What place does the first occurrence of the digit "9" (from left to right) occupy in 12,456.99?

## Section 3.2: Operations with Decimals

When performing the mathematical operations of addition, subtraction, multiplication, and division using decimals, your calculator is a great support tool. Once the given numbers are combined, rounding often comes into play when presenting the final result.

**Video Example:** Use your calculator to compute each of the following and round to the nearest hundredths place.

a. $4.32 \bullet 1.37$

b. $-523.14 \div 23.56$

c. $3.7241 + 4.35 \bullet 21.72 - 0.03$

d. $250(1.0854)^{18}$

e. $102(1+.0565)^{(2 \bullet 12)}$

f. $25476\left(1+\frac{.032}{12}\right)^{(12 \bullet 30)}$

**You Try:**

Use your calculator to compute each of the following and round to the nearest thousandths place. Check your answers in Appendix A.

a. $-5.143 \bullet 3.02$

b. $4.7345 \div 2.134$

c. $(6.41)^2 - 5.883 \div 2.17$

d. $45(2.0324)^5$

e. $207(1+.0425)^{(3 \bullet 4)}$

f. $34256\left(1+\frac{.012}{4}\right)^{(4 \bullet 20)}$

## Section 3.3: Scientific Notation

Scientific Notation is the way that scientists easily handle writing very large numbers or very small numbers in a more compact form. Below are some examples of numbers in Scientific Notation, Standard Form and Common Calculator Form. The videos below explain in more detail the process to move between forms.

| Scientific Notation | Standard Form | Common Calculator Form |
|---|---|---|
| $3.21 \times 10^{7}$ | 32,100,000 | 3.21E7 |
| $1.052 \times 10^{4}$ | 10,520 | 1.052E4 |
| $4.05 \times 10^{-6}$ | 0.00000405 | 4.05E-6 |
| $1.283 \times 10^{-3}$ | 0.001283 | 1.283E-3 |

**Video Example:**

a. Write the following numbers in standard form.

i. $5.9 \times 10^{5}$ ii. $8.3 \times 10^{-3}$

b. Write the following numbers in scientific notation.

i. 8,140,000 = ii. 0.000091 =

c. Write the following calculator forms in scientific notation and in standard form.

i. 3.723E4 ii. 8.17E-5

**You Try:**

a. Write the following numbers in standard form. Check your answers in Appendix A.

i. $3.82 \times 10^{6}$ ii. $5.03 \times 10^{-4}$

b. Write the following numbers in scientific notation.

i. 0.00000016 = ii. 5,340,000

c. Write the following calculator forms in scientific notation and in standard form.

i. 6.05E-4 ii. 1.07E5

## Section 3.4: Dollars and Cents – Working with Money

You work with decimals probably every day and may not realize you are doing so. Whenever you pay a bill or make a purchase, you are utilizing decimals. If I see the monetary amount, \$25.13, then I know to read that as "twenty-five dollars and thirteen cents". The part to the left of the decimal, i.e. the whole number part, identifies the number of dollars. The part to the right of the decimal identifies the number of cents.

**Video Example:**

a. Write each of the monetary phrases as a number.

i. Twelve dollars and seventy-five cents

ii. Thirty-two cents

iii. One hundred dollars and seven cents

iv. Five cents

b. Round each of the following monetary amounts as indicated:

i. \$127.56 to the nearest dime

ii. \$127.56 to the nearest dollar

iii. \$127.56 to the nearest ten dollars

iv. \$127.56 to the nearest hundred dollars

**You Try:**

Complete each of the following. Check your answers in Appendix A.

a. Write as a decimal: Twenty dollars and five cents

b. Round \$311.58 to the nearest dollar.

## Section 3.5: Estimating with Decimals

While using a calculator is a good way to get accurate results, there are times when we might need to quickly estimate or use "mental math" just to get a quick sense of the numbers in a given situation.

**Video Example:** Estimate and use mental math (i.e. no calculator) to quickly add the numbers below. Check your results using your calculator and see how close your estimate was.

a. 5.69, 12.456, 6.134

b. 4.312, 7.8912, 12.0145, 5.67

c. $3.055, -2.345, 1.45, -5.674$

d. $-25.15, 14.12, 11.657, -5.67$

**You Try:** Estimate and use mental math (i.e. no calculator) to quickly add the numbers below. Check your results using your calculator and see how close your estimate was. Check your answers in Appendix A.

a. $12.565, 2.345, 1.45, 5.674$

b. $-2.65, 4.92, -1.657, 5.03$

## Section 3.6: Applications with Decimals

The problem-solving process we learned in Section 1.8 applies to any application problem you are trying to solve, including those that involve decimals. Use that process to help you in this section.

**Video Example:**

a. In preparation for mailing a package, you place the item on your digital scale and obtain the following readings: 6.51 ounces, 6.52 ounces, and 6.60 ounces. What is the average of these weights? Round to the nearest hundredth of an ounce.

b. Rally went to Target with $40 in his wallet. He bought items that totaled $1.45, $2.15, $7.34, and $14.22. As he is standing in line, he wants to be sure he will have enough to purchase the items in his basket. Estimate his purchases and how much he will have left over. If tax on the items is $2.26, will he still have enough cash to pay for everything?

**You Try:** Sarah Smartshopper receives a discount of $0.10 per gallon of gas for every $100 she spends at the grocery store in a given month. During October of last year, she spent $45.23, $102.34, $13.67, $34.56, $48.72, and $52.12 at the grocery store. What will her gas discount be for October?

## Section 3.7: Decimals & Fractions

Decimals are really fractions "in disguise", as you will see in the examples below.

**Video Example:**

a. Write the decimal name in words. Then, change each to a simplified fraction.

i. 0.25 ii. 1.15 iii. 0.125

b. Change each of the following fractions to a decimal using your calculator as needed. Round to the hundredths place.

i. $\frac{3}{4}$ ii. $\frac{1}{3}$ iii. $3\frac{1}{2}$

**You Try:**

a. Write the decimal name in words. Then, change each to a simplified fraction. Check your answers in Appendix A.

i. 5.375 ii. 0.025 iii. 0.625

b. Change each of the following fractions to a decimal using your calculator as needed. Round to the hundredths place. Check your answers in Appendix A.

i. $\frac{2}{3}$ ii. $1\frac{1}{4}$ iii. $\frac{9}{10}$

## Section 3.8: Ordering Decimals & Fractions

When given numbers in decimal and/or fraction form, including those that are negative, can you order them correctly from smallest to largest? The following examples will explain ways to do that.

**Video Example:** Order each of the following sets of numbers from smallest to largest.

a. 0.042, 0.420, 0.402

b. $1.73,\ 1\frac{11}{15},\ 1.7$

c. $-2.5, -2\frac{1}{5}, -2.05, -2.55, -2.555$

**You Try:**

Order each of the following sets of numbers from smallest to largest. Check your answers in Appendix A.

a. $3.055, 3\frac{5}{9}, 3.55, 3.5, 3.555, 3.05$

b. $-1\frac{1}{4}, -1.26, -1.205, -1.27, -1.255$

## Section 3.9: Fractions, Decimals and Percents

Numbers written in percent (%) form represent amounts out of 100. The word "percent" literally means "per 100".

**Video Example:**

a. On a recent plane flight, 80% of the passengers brought devices to connect to the Internet during their flight. Represent this quantity as a fraction. If there were 200 people on the flight, how many of them brought devices to connect to the Internet?

b. In a bag of popped popcorn, 1 kernel out of every 50 were not popped. Represent this quantity as a fraction then interpret it as a percent. If there were 500 kernels in the bag, how many of them were not popped?

Decimals, fractions, and percents are closely connected. The following table shows how to convert from one to the other.

| Direction of Conversion | Example | Process |
|---|---|---|
| Percent to Decimal | $50\% = \frac{50}{100} = 0.50$ | Remove % sign.<br>Place over 100.<br>Divide to get decimal. |
| Percent to Fraction | $50\% = \frac{50}{100} = \frac{1}{2}$ | Remove % sign.<br>Place over 100.<br>Reduce fraction. |
| Decimal to Percent | $0.50 = 50\%$ | Multiply by 100.<br>Include % sign. |
| Fraction to Percent | $\frac{1}{2} = .50 = 50\%$ | Divide.<br>Multiply by 100.<br>Include % sign. |

**Video Example:** Complete the missing parts of the table. Simplify all fractions.

| Fraction | Decimal | Percent |
|---|---|---|
| | | 10% |
| | 0.04 | |
| $\frac{3}{4}$ | | |
| | 0.625 | |
| | | 240% |

**You Try:** Work each of the following. Check your answers in Appendix A.

a. At a local community college, 75 of the 100 students in the film program had already made their own short films. Write this quantity as a %.

b. In a bag of 25 pennies, 5 of them were made prior to 1980. Represent this quantity as a fraction then interpret as a percent.

c. Complete the missing parts of the table. Simplify all fractions.

| Fraction | Decimal | Percent |
|---|---|---|
| $\frac{1}{8}$ | | |
| | O.0625 | |
| | | 80% |
| | | 125% |

## Section 3.10: Percent Equations

When working with a problem involving percents, the most straightforward way to solve it is by setting up a *percent equation*. Every percent equation will take the basic form shown below.

$$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$$

Let's see how this equation will help us solve percent problems.

**Video Example:** Determine the missing number in each of the following. Round to two decimals as needed.

a. What is 10% of 20?

b. 25 is 12.25% of what number?

c. 12% of 600 is what number?

d. 15 out of 35 is what %?

**You Try:** Determine the missing number in each of the following. Round to two decimal places as needed. Check your answers in Appendix A.

a. What is 15% of 324?

b. 132 is 25.12% of what number?

c. 10% of 457 is what number?

d. 14 out of 28 is what %?

## Section 3.11: Applications with Percents and Decimals

The problem-solving process we learned in Section 1.8 applies to any application problem you are trying to solve, including those that involve percents. Use that process to help you in this section.

**Video Example:**

a. Trader Joe's sold 4,872 bags of tortilla chips last month. If 1,643 of these bags were fat free, find the percent that were fat free. Round your answer to the nearest hundredth percent.

b. A salesperson receives a 6.9% commission on her sales. If her total sales for the month are $4,100, what is her commission?

c. You and your friends spend $56.45 at a restaurant. Usually, tips are around 20% of the bill. However, with the outbreak of COVID-19, you decided to tip 25%. How much greater is the bill with 25% tip as opposed to 20% tip?

d. A shirt was originally priced at $36.12. The store is having a 20% off sale. How much will you pay for the shirt after the discount (not including tax)?

**You Try:** Work each of the following. Check your answers in Appendix A.

a. To win the election as president of the United States of America, a person must obtain 270 out of 538 possible votes from the Electoral College. What percentage of the overall electoral votes is this? Round your answer to the nearest tenth of a percent.

b. The sales tax rate for the state of Washington was 9.3%. What is the final cost of a $5,300 car in Washington, including tax? Round your answers to the nearest cent as needed.

c. A violin was originally priced at $350. The store is having a 40% off sale. How much will you pay for the violin after the discount is applied (not including tax)? Round your answer to the nearest cent.

## Lesson 3 – Practice & Review

1.

a. Write the number 134.546 using words then round to the hundredths place. What place does the digit “5” occupy in 134.546?

b. Write the number 0.012 using words then round to the hundredths place. What place does the digit “2” occupy in 0.012?

c. Write the number 475.63 using words then round to the hundreds place. What place does the digit “7” occupy in 475.63?

d. Write the number 13,356.899 using words then round to the hundredths place. What place does the digit “5” occupy in 13,356.899?

2. Use your calculator to compute each of the following and round to the nearest hundredths place.

a. $5.3 \bullet 2.14$ b. $-415.24 \div 12.63$ c. $5.41 + 2.35 \bullet 2.72 - 0.02$

d. $50(2.065)^{13}$ e. $201(1+.0655)^{(6\bullet 12)}$ f. $4725\left(1+\frac{.015}{12}\right)^{(12\bullet 15)}$

3. a. Write the following numbers in standard form.

i. $4.145 \times 10^{3}$ ii. $1.611 \times 10^{-3}$

b. Write the following numbers in scientific notation.

i. 0.000252 = ii. 312,000

c. Write the following calculator forms in scientific notation and in standard form.

i. 7.12E3 ii. 1.07E-2

4.
a. Write each of the monetary phrases as a number.

i. Twenty-five dollars and sixty cents

ii. Fifty-five cents

iii. Three thousand twelve dollars and four cents

iv. Fifteen cents

b. Round each of the following monetary amounts as indicated:

i. \$3,214.56 to the nearest dime

ii. \$3,214.56 to the nearest dollar

iii. \$3,214.56 to the nearest ten dollars

iv. \$3,214.56 to the nearest hundred dollars

5. Estimate and use mental math (i.e. no calculator) to quickly add the numbers below. Check your results using your calculator and see how close your estimate was.

a. 6.78, 10.01, 4.32, 5.75

b. 5.489, 9.1912, 11.541, 7.565

c. $-5.033, -1.543, 4.15, -6.574$

d. $35.02, 10.12, -12.657, -4.67$

6.
a. You recently purchased ground beef at the grocery store in the following amounts: 2.05 lbs, 2.67 lbs, 2.98 lbs, and 3.15 lbs. What was the average weight between the four packages of ground beef? Round to the nearest hundredth of a pound.

b. Your aunt gave you a $100 gas card. You bought gas three times in the amounts of $25.76, $32.13 and $35.45. How much gas can you purchase using the card on your next trip to the gas station?

c. Jamie takes a summer job hoping to save enough to buy a car that costs $5000. Her job pays $14.75 per hour. If she plans to work 10 weeks during the summer, how many hours will she need to work each week to save enough money to buy the car?

7.
a. Write the decimal name in words. Then, change each to a simplified fraction.

i. 4.14 ii. 0.375 iii. 2.125

b. Change each of the following fractions to a decimal using your calculator as needed. Round to the hundredths place.

i. $\frac{12}{7}$ ii. $\frac{7}{9}$ iii. $4\frac{1}{4}$

8. Order each of the following sets of numbers from smallest to largest.

a. 0.021, 0.21, 0.022, 0.22

b. $2.26,\ 2\frac{1}{4},\ 2.3, 2.12$

c. $-3.09, -3\frac{1}{10}, -3.11, -3.101$

9. Complete the missing parts of the table. Simplify all fractions.

| Fraction | Decimal | Percent |
|---|---|---|
| | | 15% |
| | 0.06 | |
| $\frac{15}{8}$ | | |
| | 0.10 | |
| | | 150% |

10. Determine the missing number in each of the following. Round to two decimal places as needed.

a. What is 15% of 40?

b. What is 20% of 21.56?

c. 14 out of 32 is what %?

d. 23 out of 61 is what %?

e. 25% of 500 is what number?

f. 15 is what % of 71?

g. 42 is what % of 31?

h. 14 out of 28 is what %?

11. Work each of the following. Round to the nearest hundredth as needed.

a. In a recent election, 1,234 of the 2,575 votes were for Mayor Blume. What % of the votes did Mayor Blume receive?

b. Blane worked at a local restaurant and earned 5% of his wages from tips. If he earned $212.34 in wages in a given week, how much did he receive in tips?

c. Cindy received a job when she graduated from college and her starting salary was $52,345 per year. She was told that if she did well in her first year, she could expect to receive 105% of this salary to start the next year. What could she expect to receive as a salary during her second year assuming she did well in her first year?

d. Spencer's recently advertised a washer on a sale of 15% off. If the original price of the washer was $1299.86, what is the sale price?

# LESSON 4 – RATIOS, PROPORTIONS & ALGEBRAIC EXPRESSIONS

*Shelby is working to set up a garden bed on her organic farm and is comparing bulk dirt prices for local sellers. Dirt Dobbers sells loads of organic soil at a cost of $215 per dump truck. Each dump truck load delivers 5 cubic yards. Cathy's Compost sells the same soil at a cost of $350 but for 7 cubic yards. Which seller has the better price per cubic yard?
*[Answer: Dirt Dobbers at $43 per cubic yard]*

In order to answer this question, you will need to understand how to work with ratios and unit rates. In this lesson, you will learn all about working with ratios and rates as well as proportions and algebraic expressions.



** Image source (public domain images):*
[https://freesvg.org/harvestable-resources-garden-new](https://freesvg.org/harvestable-resources-garden-new)

## Section 4.1: Ratios and Rates

To compare two different quantities, we write what is called a *ratio*. Units are critical and are always included if they are present to begin with although if your units are the same, they will "cancel out". The examples below demonstrate the different notations you may see when writing ratios.

**Video Example:**
Write each of the following as a ratio in simplest form.

a. 8 feet to 16 feet

b. 6:4

If the quantities you are comparing have different units, then your ratio is known as a *rate.* Units are especially important here and should absolutely be included. The units will not "cancel out" of a rate.

**Video Example:**
Write each of the following as a ratio in simplest form.

a. 12 miles in 10 hours

b. 15 peanuts to 20 almonds

**You Try:** Use the information to write each ratio in simplest form. Indicate if the ratio is also a rate. Check your answers in Appendix A.

a. 5 feet:10 feet

b. 12 geese to 15 ducks

## Section 4.2: Unit Rates

A *unit rate* is a special kind of rate in which the denominator of the ratio is equal to 1. This kind of rate allows for direct comparison of different rates as seen in the examples below. As with rates, units are essential and must be included.

**Video Example:**
a. Write each of the following as a unit rate:

i. 5280 feet in a mile  ii. 60 seconds per each minute  iii. Gasoline at $2.49 a gallon

b. Use unit rates to compare the quantities in each case below.

i. Which is faster, 12 miles in 10 hours or 10 miles in 8 hours?

ii. Determine which chips are the better buy, $4.99 for 20.50 oz. or $4.29 for 12.50 oz.

**You Try:** Amazon.com recently advertised the following choices for ibuprofen tablets (200mg). Use unit rates to determine which is the better buy. Check your answers in Appendix A.

Option 1: 360 pills for $15.45  Option 2: 300 pills for $12.98

## Section 4.3: Proportions

Earlier in the lesson, we were given the ratio, 12 miles in 10 hours, which we simplified to 6 miles in 5 hours. We can write that in a formal mathematical way as follows:

$$\frac{12 \text{ miles}}{10 \text{ hours}} = \frac{6 \text{ miles}}{5 \text{ hours}}$$

The statement above is called a *proportion* because it sets two ratios equal to each other. Because the above ratios are equivalent, the equality statement is true.

Suppose, however, that the following problem was posed:

> "If George walks 6 miles in 5 hours, how far would he walk in 10 hours?"

We set up the following *proportion.* Note that the unknown is labeled as x which is called a *variable*. We will discuss variables more later in this lesson, but we can begin to use them now.

$$\frac{x \text{ miles}}{10 \text{ hours}} = \frac{6 \text{ miles}}{5 \text{ hours}}$$

The distance George walks in 10 hours is our unknown value and is represented by the variable x. Technically, in this problem, we know that our solution for x is 12. But how would we determine that mathematically? First, because our ratios of units are the same (miles/hours) we can simplify our statement this way:

$$\frac{x}{10} = \frac{6}{5}$$

Then, we can use what is called a *cross-product* to rewrite as follows:

$$\frac{x}{10} \quad \frac{6}{5}$$

Multiply across
the = sign
bottom to top

$$x = \frac{6 \cdot 10}{5}$$

And finally, we can write our final solution as $x = \frac{60}{5} = 12$.

The final answer to our original question, "if George walks 6 miles in 5 hours, how far would he walk in 10 hours" is that George could walk 12 miles in 10 hours. We solved this problem using *proportional reasoning,* one of the most-used problem-solving techniques in mathematics.

The following examples will illustrate additional ways to work with and solve proportions using the *cross-product method*.

**Video Example:**
Use the cross-product method to determine the value for the variable in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{3}{4}=\frac{t}{40}$

b. $\frac{r}{5}=3$

c. $\frac{6}{12}=\frac{18}{x}$

d. $\frac{2.3}{x}=\frac{4.1}{5.6}$

**You Try:**
Use the cross-product method to determine the value for the variable in each of the following proportion problems. Round any decimals to the hundredths place.
Check your answers in Appendix A.

a. $\frac{x}{12}=\frac{3}{6}$

b. $\frac{6}{5}=\frac{10}{p}$

## Section 4.4: Applications with Proportions

The problem-solving process we learned in Section 1.8 applies to any application problem you are trying to solve, including those that involve proportions. Use that process to help you in this section.

**Video Example:**

a. Ten gallons of water leak from a hose in 1 day. At this rate, how much water will leak from the hose in 9 days?

b. Mary earned $112.50 last week working 12 hours at her part-time job. If she wants to earn $175 the next week and is paid at the same rate, how many hours would she need to work?

**You Try:**

In November 2012 President Obama visited Phnom Penh, Cambodia as part of a summit of Asian leaders. Traffic in the city came to almost a complete standstill with cars moving at a rate of 2 miles in 4 hours. At this rate, how long would it take to travel a distance of 3.5 miles? Check your answers in Appendix A.

## Section 4.5: Writing Algebraic Expressions

A *variable*, usually represented by a letter or symbol, is a unknown quantity in a mathematical equation or expression. We worked with variables a little in section 4.3 and 4.4. We will learn more about them in this section.

An *algebraic expression* is a mathematical statement that can contain numbers, variables, and operations (addition, subtraction, multiplication, division, etc…).

**Video Example:**

a. Juan is 6 inches taller than Niko. Let N represent Niko's height in inches. Write an algebraic expression to represent Juan's height.

b. Juan is 6 inches taller than Niko. Let J represent Juan's height in inches. Write an algebraic expression to represent Niko's height.

c. Suppose sales tax in your town is currently 9.8%. Write an algebraic expression representing the sales tax for an item that costs D dollars.

d. You started this year with $362 saved and you continue to save an additional $30 per month. Write an algebraic expression to represent the total amount saved after *m* months.

e. Movie tickets cost $8 for adults and $5.50 for children. Write an algebraic expression to represent the total cost for *A* adults and *C* children to go to a movie.

**You Try:**
Work each of the following. Check your answers in Appendix A.

a. There are about 80 calories in one chocolate chip cookie. If we let *n* be the number of chocolate chip cookies eaten, write an algebraic expression for the number of calories consumed.

b. Brendan recently hired a contractor to do some necessary repair work. The contractor gave a quote of $450 for materials and supplies plus $38 an hour for labor. Write an algebraic expression to represent the total cost for the repairs if the contractor works for *h* hours.

c. A concession stand charges $3.50 for a slice of pizza and $1.50 for a soda. Write an algebraic expression to represent the total cost for *P* slices of pizza and *S* sodas.

## Section 4.6: Evaluating Algebraic Expressions

Algebraic expressions are the building blocks of algebraic equations. Equations are what we often use to set up and solve real-world problems. We often are asked to evaluate expressions at particular values so practice with this skill is important.

**Video Example:**
For a – i below, find the value of each expression when $w = 2$. Simplify your answers.

a. $w-6$

b. $6-w$

c. $5w-3$

d. $w^3$

e. $3w^2$

f. $(3w)^2$

g. $\frac{4}{5w}$

h. $\frac{5}{4}w$

i. $3^w$

j. Evaluate $ab+c$ given a = -5, b = 7 and c = -3.

k. Evaluate $a^2-b^2$ given a = -5 and b = -3.

l. A local window washing company charges $11.92 for each window plus a reservation fee of $7. The total cost for washing *w* windows is given by $11.92w + 7. Use this expression to determine the total cost for washing 17 windows.

**You Try:**
Work each of the following. Check your answers in Appendix A.

a. Evaluate $b^2 - 4ac$ given a = 5, b = -1, and c = 2.

b. Evaluate $3a^2 - b^2$ given a = -1 and b = -2.

c. A concession stand charges $3.50 for a slice of pizza and $1.50 for a soda. The total cost for *P* slices of pizza and *S* sodas is represented by the algebraic expression 3.50*P* + 1.50*S*. What is the cost for 5 slices of pizza and 3 sodas?

d. Suppose sales tax in your town is currently 9.8%. The sales tax for an item that costs *D* dollars is represented by the algebraic expression 0.098*D*. What is the sales tax on an item that costs $101.21? Round to the nearest cent as needed.

## Section 4.7: Simplifying Algebraic Expressions

Understanding how algebraic expressions are constructed will help us when solving applications problems and setting up expressions from scratch.

T*erms* are the building blocks of algebraic expressions and are joined together through addition or subtraction. The example below presents important definitions and vocabulary that are used to discuss terms and expressions.

**Video Example:**

a. Consider the algebraic expression $4x^5 + 3x^4 - 22x^2 - x + 17$. Identify each term along with its coefficient.

b. Consider the algebraic expression $-4m^2 - m + \frac{1}{2}m^3 - \frac{m}{3} + 4$. Identify each term along with its coefficient.

*Like terms* are terms whose variable components are exactly the same. The video example shows how to combine like terms to simplify an expression.

**Video Example:**

Combine like terms to simplify each expression as much as possible.

a. $4x^5 + 3x^5 - 2$

b. $3a - 6a + 10a - a$

c. $5x - 10y - 6x - 10y$

d. $7n + 3n^2 - 2n^3 + 8n^2$

e. $\frac{1}{2}x - x$

f. $\frac{w}{3} - 2w^2 + 4w + 3$

**You Try:**

Work each of the following. Check your answers in Appendix A.

a. For each algebraic expression below, identify each term along with its coefficient.

i. $2x^5 - 3x^3 - 2x^5 + 7$   ii. $2b - 5b + 7b - b$   iii. $\frac{2y}{3} + 3y^2 - 3y + 5$

b. Combine like terms to simplify each expression as much as possible.

i. $2x^5 - 3x^3 - 2x^5 + 7$   ii. $2b - 5b + 7b - b$   iii. $\frac{2y}{3} + 3y^2 - 3y + 5$

## Section 4.8: The Distributive Property

In the previous section, we were able to easily identify terms and coefficients because the expressions we dealt with were already "spread out" and their terms were connected by only addition or subtraction. What happens if the expression initially involves multiplication and the use of ()? We will need to apply a very powerful mathematical property called the *Distributive Property* which says:

$$a(b+c)=ab+ac$$

The examples below explain in more detail how this property is applied.

**Video Example:**
a. Use the Distributive Property to expand each of the following expressions.

i. $5(2x+4)$ ii. $-(b-1)$ iii. $-3(x^2-2x+7)$

b. Use the Distributive Property to expand where needed and combine like terms to simplify each of the following expressions.

i. $5(2x+4)-(x-1)$ ii. $-2(b-1)+(b-2)$ iii. $-3(x^2-2x+7)+4(x^2-5x)$

**You Try:**
Use the Distributive Property to expand where needed and combine like terms to simplify each of the following expressions. Check your answers in Appendix A.

a. $-2(x-1)+(3x-4)$ b. $3(w-1)-(w-4)$ c. $-2(x^2-4x+3)+2(x^2-3x)$

## Section 4.9: Applications with Algebraic Expressions

**Video Example:**

a. The algebraic expression, $0.8(220-A)$, gives the *recommended* maximum heart rate, in beats per minute, for a person who is *A* years of age. What is the recommended maximum heart rate for a person who is 40 years old?

b. Sally buys a $1,000 savings bond that pays 4% simple interest each year. The accrued value of her investment *P*, after *t* years at interest rate *r* in decimal form, is given by the algebraic expression: $P+\mathrm{Pr}t$. How much will the bond be worth after 5 years?

c. The algebraic expression, $266(1.009)^t$, estimates the population of the United States (in millions of people), t years after 1995. Use this formula to estimate the U.S. population in 1995. Then, use this formula to estimate the U.S. population in 2016. Round each answer to the nearest million people.

**You Try:**

Work each of the following. Check your answers in Appendix A.

a. Paul is planning to sell bottled water at the local carnival. He buys 2 crates of water (2000 bottles) for $360 and plans on selling the bottles for $1.50 each. Paul's profit in dollars, from selling *b* bottles of water, is given by the algebraic expression $1.5b - 360$. Determine Paul's profit if he sells all 2000 bottles of water.

b. A golfer strikes a golf ball. The height, in feet, of the ball after *t* seconds is given by the algebraic expression $-16(t^2 - 5t)$. Determine the height of the ball after 3 seconds.

Lesson 4 – Practice & Review

1. Write each of the following as a ratio in simplest form. Indicate if the ratio is also a rate.

a. 4 inches to 2 feet

b. 7:10

c. 225 miles in 3 hours

d. 9 to 27

e. 12 feet:36 feet

f. 10 tablespoons to 2 cups

2. Convert each of the following into a unit rate:

a. 15 feet in 2 minutes

b. 11 gallons in 2 hours

c. $725 in 5 hours

d. 72 miles to 3 gallons

e. $25.24 for 4 bottles

f. 40 seeds to 20 pots

3. Use unit rates to compare and determine which is the better value. Round your answers to two decimal places.

a. Apple-cider Vinegar, 128 ounces for $6.99 or 32 ounces for $2.49

b. Bread, 24 ounces for $4.99 or 18 ounces for $4.29

c. Milk, 128 ounces for $4.58 or 52 ounces for $3.98

d. Vitamin E, 400IU 100-count for $19.99 or 400IU 50-count or for $8.99

4. Use the cross-product method to determine the value for the variable in each of the following proportion problems. Round any decimals to the hundredths place.

a. $\frac{x}{14}=\frac{6}{7}$

b. $\frac{12}{72}=\frac{x}{12}$

c. $\frac{5}{61}=\frac{45}{b}$

d. $\frac{16}{24}=\frac{4}{r}$

e. $\frac{x}{-6}=\frac{36}{-54}$

f. $\frac{18}{24}=\frac{6}{-y}$

g. $\frac{3.1}{a}=\frac{1.2}{2.5}$

h. $\frac{-x}{4.2}=\frac{-3.5}{4}$

i. $\frac{4}{y}=5$

j. $\frac{x}{10}=3$

5. Solve each of the following application problems using proportions. Round all answers to the hundredths place as needed.

a. Vets will often prescribe daily Benadryl for dogs at a rate of 4 mg per 2 kg of body weight. If your dog weighs 40 kg, what is the max mg of Benadryl your vet might prescribe per day?

b. Matt weighed himself today and found that he had lost 10 pounds since he weighed 3 weeks ago. What was his weight loss in pounds per week?

c. Your keto diet plan allows you to eat 50 grams of carbohydrates a day. You want to incorporate vanilla ice cream into your diet plan and, when reading the ice cream label at the grocery store, you see that each serving of vanilla ice cream contains 15 grams of carbohydrates. How many servings of vanilla ice cream could you eat in one day on your plan?

d. Brian is concerned about getting enough fiber in his diet. He loves eating oatmeal for breakfast. If 1 oz of oatmeal (dry) gives about 3 g of fiber, and he needs to eat about 38 g of fiber per day, how many ounces of oatmeal would he need to eat to meet his daily fiber needs?

e. Brenda is taking a multi-day trip and wants to drive a total of 500 miles per day. If it takes her 4 hours to travel 200 miles on a given day, how long will it take her each day to drive 500 miles?

6.

a. Rick and Sarah went out to a seafood restaurant. She ordered salmon and he ordered steak. The salmon cost $10 more than the steak. Let *S* represent the cost of the salmon. Write an algebraic expression for the cost of the steak.

b. Brianna plays for her college soccer team. She has in her closet three times the number of uniform shirts as pairs of shorts. If she has *S* pairs of shorts, write an expression for the number of uniform shirts she has.

c. Rodney earns 15% commission on every dollar of product that he sells. If he sold D dollars of product last month write an algebraic expression for the commission that he earned.

d. Fritz made purchases at a store with sales tax rate of 7.2%. If his purchases totaled T dollars, write an expression to determine the amount of sales tax on that purchase.

7. For a – i below, find the value of each expression when $x$ = -3. Simplify your answers.

a. $x-2$

b. $6-x$

c. $3x-7$

d. $x^2$

e. $-2x^3$

f. $(2x)^2$

g. $\frac{3}{5x}$

h. $\frac{2}{3}x$

i. $3^{-x}$

8.

a. Evaluate $ab+c$ given a = 4, b = -7 and c = -2.

b. Evaluate $a^2-b^2$ given a = -3 and b = 2.

c. Evaluate $2a^2-b$ given a = -2 and b = 1

d. Evaluate $ab-c$ given a = -2, b = 3 and c = -1.

9.
For each algebraic expression below, identify each term along with its coefficient.

a. $-3y^5+\frac{1}{4}y^4-2y^4+6$  b. $3a-6b+2a-7b$  c. $\frac{x}{2}-2x^2+y+4$

10. Combine like terms to simplify each expression as much as possible.

a. $-3y^5+\frac{1}{4}y^4-2y^4+6$  b. $3a-6b+2a-7b$

c. $\frac{x}{2}-2x^2+x+4x^2-1$  d. $4x-3xy+2y-x+y$

11. Use the Distributive Property to expand each of the following expressions.

a. $-2(2y+3)$  b. $-(x+3)$  c. $\frac{1}{3}(a^2-3a+4)$

12. Use the Distributive Property to expand where needed and combine like terms to simplify each of the following expressions.

a. $-2(x+1)-(2x-3)$  b. $-(b-3)+(2b-3)$  c. $-(x^2+2x-7)+4(-x^2-5x)$

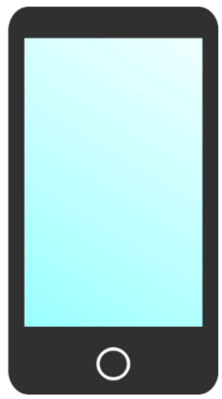
13. Work each of the following rounding to two decimal places as needed.

a. Rick and Sarah went out to a seafood restaurant. She ordered salmon and he ordered steak. They decided to leave a 25% tip. The total bill, including tax and tip, can be determined using the expression $1.25(2K + 10)$ where $K$ is the cost of Rick's steak. If Rick's steak cost $21.52, what was the total bill?

b. Rodney earns 15% commission each month on every dollar of product that he sells. He also earns a flat wage of $3000 each month in addition to his commission. If he sold $D$ dollars of product last month, his wages can be determined using the expression $3000 + 0.15D$. What are his wages if he sells $45,000 in product?

# LESSON 5 – EQUATIONS

*Tirana bought a cell phone from a new cell phone company. She signed up for a rate plan that would charge her $51 per month plus 8 cents per minute. If she budgeted $100 a month for her total cell phone bill, how many minutes can she talk each month? *[Answer: 612 minutes and 30 seconds]*



In order to answer this question, you will need to understand how to set up and solve a basic mathematical equation. In this lesson, you will learn all about algebraic equations including solving them and setting them up to solve application problems.



** Image source (public domain images):*
https://freesvg.org/fx13-phone-1

## Section 5.1: Algebraic Equations

An *algebraic equation* is a mathematical sentence stating that an algebraic expression is *equal to* a specified value, variable, or another expression. The *solution* to an algebraic equation is the value, or values, that make the equation true.

**Video Example:**

a. Verify that $x = -3$ is a solution to the algebraic equation $5x - 2 = 8x + 7$.

b. Is $m = -1$ a solution to the algebraic equation $m + 9 = 3m + 5$?

c. Is $a = 4$ a solution to the algebraic equation $0.50a - 1.6 = 0.4$?

**You Try:**

Work each of the following. Check your answers in Appendix A.

a. Verify that $x = -1$ is a solution to the algebraic equation $2(x + 2) = 2$.

b. Is $t = 4$ a solution to the algebraic equation $t + 2 = -3t - 7$?

## Section 5.2: Solving One-Step Equations (part 1)

In order to identify the solution to an algebraic equation, we use a process called *isolating the variable*. To isolate a variable, we use properties of equality to create a sequence of equivalent equations ending with the variable by itself on one side of the equation and everything else on the other.

The first property of equality we will study is the *Addition/Subtraction Property of Equality* which says:

If a = b, then a + c = b + c. If a = b, then a – c = b – c.

This property is explored in more detail in the examples below.

**Video Example:**
Solve for the variable in each of the following equations. Check your answers.

a. $x+1=4$

b. $s-4=2$

c. $x+7=18$

d. $r-4.5=-5.1$

e. $-4+b=45$

f. $3=19+m$

**You Try:**
Solve for the variable in each of the following equations. Check your answers as in the video then check Appendix A as well.

a. $x+3=5$

b. $w-3=6$

c. $-5.2+v=14.3$

d. $x-6=-3$

## Section 5.3: Solving One-Step Equations (Part 2)

The second property of equality we will study is the *Multiplication/Division Property of Equality* which says:

If $a = b$, then $a \cdot c = b \cdot c$.  If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$

This property is explored in more detail in the examples below.

**Video Example:**
Solve for the variable in each of the following equations. Check your answers.

a. $2x = 4$

b. $\frac{y}{3} = 4$

c. $-x = 4$

d. $-4.4r = 5.3$

e. $-3y = 42$

f. $-5 = \frac{5}{6}x$

**You Try:**
Solve for the variable in each of the following equations. Round to the nearest hundredth as needed. Check your answers as in the video then check Appendix A as well.

a. $3x = -9$

b. $\frac{t}{2} = 1$

c. $-3.2 = \frac{w}{5}$

d. $\frac{3}{5}n = -2$

## Section 5.4: Solving Multi-Step Equations (Part 1)

The equations we looked at in sections 5.2 and 5.3 could be solved in one step. Most of the equations that we experience will require more than one step to solve. The process below will generally work to solve equations that require more than one step:

1. Apply the Addition/Subtraction Property of Equality.
2. Apply the Multiplication/Division Property of Equality to isolate the variable.
3. Check by substituting your answer into the original equation.

**Video Example:**
Solve for the variable in each of the following equations. Check your answers.

a. $2b-4=12$

b. $4+3r=5$

c. $3.2=19.1-2m$

d. $11-y=32$

e. $3+\frac{3}{5}x=12$

f. $5-2x=4$

**You Try:**
Solve for the variable in each of the following equations. Round to the nearest hundredth as needed. Check your answers as in the video then check Appendix A as well.

a. $3x-14=40$

b. $\frac{3}{4}w-8=-2$

c. $14.2=2.1-x$

d. $-3-w=-4$

## Section 5.5: Solving Multi-Step Equations (Part 2)

Often, equations will contain parentheses and like terms must be combined before we can isolate the variable. The process below will generally work with equations of this type.

1. Simplify each side of the equation. Remove parentheses if necessary. Combine like terms.
2. Apply the Addition/Subtraction Property of Equality so that all terms containing the variable are on one side and all non-variable terms are on the other side.
3. Simplify each side of the equation by combining like terms.
4. Apply the Multiplication/Division Property of Equality to isolate the variable.
5. Check by substituting your answer into the original equation.

**Video Example:**
Solve for the variable in each of the following equations. Check your answers.

a. $x-5=4x+7$

b. $-(x-2)=15$

c. $3.1(4n-2.2)=5.2(n+3.4)$

d. $4-(2y-1)=2(5y+9)+y$

**You Try:**
Solve for the variable in each of the following equations. Round to the nearest hundredth as needed. Check your answers as in the video then check Appendix A as well.

a. $14=2-x$

b. $-3-\frac{2}{3}w=-4$

c. $m-5=8m+2$

d. $-2.1(5x-12)=5.4x-6$

## Section 5.6: Applications with Equations (Part 1)

The problem-solving process we learned in Section 1.8 applies to any application problem you are trying to solve, including those that involve solving equations. Use that process to help you in this section.

**Video Example:**

a. The maximum heart rate is the highest heart rate achieved during maximal exercise. In general, you get the most benefits and reduce the risks when you exercise within your *target* heart rate zone. Usually this is when your exercise heart rate (pulse) is about 80 percent of your maximum heart rate. The formula $M = 0.8(220 - A)$, gives the *recommended* maximum heart rate, *M*, in beats per minute, for a person who is *A* years of age. What is the age of a person whose maximum recommended heart rate is 135 beats per minute? Round to the nearest year.

b. Simple interest is given by the formula $A = P + \text{Pr}t$ where *A* is the accrued value of the investment after *t* years, and *P* is the starting principal invested at an annual percentage rate of *r,* expressed as a decimal. Sally buys a $1,000 savings bond that pays 4% simple interest each year. How long will it take for the accrued value of her investment to equal 1500.00? Round to the nearest year.

**You Try:**
Work each of the following. Round to the nearest hundredth as needed. Check your answers in Appendix A.

a. The cost of tuition at a local community college is given by the equation $C = 76n$, where C represents the total cost of tuition and *n* represents the number of credits taken. If you have \$800 to spend on tuition, how many credits can you take?

b. Paul is planning to sell bottled water at the local fair. He buys 2 crates of water (2000 bottles) for \$60 and plans on selling the bottles for \$1.50 each. Paul's profit, *P,* in dollars, from selling *b* bottles of water is given by the formula *P* = 1.5*b* – 60. How many bottles does Paul need to sell in order to break even? (Note: break-even occurs when Profit = \$0).

## Section 5.7: Applications with Equations (Part 2)

We can use our problem-solving process to address applications that involve the writing of equations from scratch. The same steps are used with the addition of a step to name any variable we choose to use in our problem. The examples below provide more details.

**Video Example:**

a. The cost of leasing a new Ford mustang is $2,311 for a down payment and processing fee plus $276 per month. For how many months can you lease this car with $10,000?

b. You just purchased an Ipad for $872. The value of the Ipad decreases by $175 per year. How long before the Ipad is worth half the original value?

c. You have taken 4 of 5 tests for your math class and earned scores of 85%, 92%, 89%, and 86%. What % must you score on the last test to achieve a 90% average overall?

**You Try:**
Work each of the following. Round to the nearest hundredth as needed. Check your answers in Appendix A.

a. Your yard is a mess, and you decide to hire a landscaper. Garden Pros charge a \$50 consultation fee plus \$36 per hour for the actual work. If the total cost is \$212, how many hours did the landscapers work?

b. When purchasing a new cell phone plan, you are given two pay options (both include unlimited text). With Plan A, you pay \$0.23 per minute. With Plan B, you pay \$0.13 per minute plus a flat fee of \$6.80. How many minutes would you have to talk each month for the plans to cost the same amount?

## Section 5.8: Literal Equations

Literal equations are equations that represent real world situations and often contain more than one variable. Below are some examples of literal equations:

- $A = \frac{1}{2}bh$ represents the area of a triangle, A, with base, *b,* and height, *h*.
- $A = P + Prt$ represents the amount accrued, A, after an annual interest rate of r (written as a decimal) is applied to an invested principal, P, for t years.
- $F = \frac{9}{5}C + 32$ provides the degrees Fahrenheit, F, given the degrees Celsius, C.

There are many, many literal equations and an important skill is to be able to rewrite these equations and isolate different variables as needed in a particular situation. Our steps for solving equations come in handy as the following examples illustrate.

**Video Example:**
Solve each equation for the indicated variable.

a. Solve for *B*: $A + B + C = D$

b. Solve for *c*: $4abc = 32$

c. Solve for *x*: $y = mx + b$

d. Solve for *b*: $y = mx + b$

e. Solve for *C*: $F = \frac{9}{5}C + 32$

f. Solve for *t*: $A = P + Prt$

**You Try:**
Solve each equation for the indicated variable. Check your answers in Appendix A.

a. Solve for *C*: $A+B+C=D$

b. Solve for *b*: $4abc=32$

c. Solve for *y*: $5x-y=2$

d. Solve for *x*: $5x-y=2$

e. Solve for *F*: $C=\frac{5}{9}(F-32)$

f. Solve for r: $A=P+\mathrm{Pr}t$

## Section 5.9: Applications with Literal Equations

When addressing applications that involve literal equations, usually, we are provided with the equation so that part of the work is done for us. What is left is to determine which variable values we are given and which are unknown. Then we solve as previously discussed. The following examples illustrate this concept.

**Video Example:**

a. In London, the high temperature on June 9, 2020 was listed as 16 degrees Celsius. Use the literal equation, $C=\frac{5}{9}(F-32)$, to determine the equivalent temperature in degrees Fahrenheit.

b. Simple interest is given by the formula $A=P+\mathrm{Pr}t$ where *A* is the accrued value of the investment after *t* years, and *P* is the starting principal invested at an annual percentage rate of *r,* expressed as a decimal. Ramón makes an initial investment of $1205 at 3.15% interest. How long should he leave the money invested if he wants to accrue $2500?

**You Try:**
Work each of the following. Round to the nearest hundredth as needed. Check your answers in Appendix A.

a. The literal equation, $D = rt$, indicates that distance traveled, D, is computed as rate of travel multiplied by time traveled. Use this equation to determine the rate of travel if a person travels 365 miles in 5 hours.

b. The literal equation, $P = 2l + 2w$, indicates that the perimeter (or distance around), P, of a rectangle is computed as twice the length, l, of the rectangle plus twice the width, w. Use this equation to determine the length of the rectangle if the perimeter is 30 inches and the width is 5 inches.

Lesson 5 – Practice & Review

1. Verify that $x=-1$ is a solution to each of the following algebraic equations.

a. $5x+8=3$

b. $-3x+9=2x+14$

2. Is $x=3$ a solution to either of the following algebraic equations?

a. $15x-2=3(x-1)$

b. $0.3x-7.5=-6.6$

3. Solve for the variable in each of the following equations. Be sure to check your final result. Round any decimals to two decimal places as needed. If a problem has fractions in it, leave your final result as a fraction.

a. $2x+(-4)=12$

b. $w-6=-3$

c. $-4.2-3x=11.1-2x$

d. $3y-2y=2y+5$

e. $\frac{x}{2}-3=3(x-4)$

f. $\frac{4}{3}n=-1$

g. $5(x-1)+4x=40$

h. $\frac{5}{4}x-10=-12$

i. $-14.2x+3=3.1-2x$

j. $-5-x=-6$

k. $-t-4=6t+3$

l. $-1.1(3x-10)=(4.1x-5)-2x$

4. Work each of the following. Round to the nearest hundredth as needed.

a. Movie tickets to your local theatre are $7.50 per person. Let N = represent the number of tickets you can purchase. If you have $100 to spend, write and solve an equation to determine how many tickets you can buy.

b. Rachel visited a local Starbucks and purchased a Grande Frappuccino beverage and 5 cake pops. Her beverage cost $5.24 and her total purchase was $17.59 (before tax). Let P represent the cost of each cake pop. Write and solve an equation to determine the price of each cake pop.

c. At a local lake, fishing boats can be rented for a $75 nonrefundable deposit and $100 per hour. You and your friends spent $675 to rent the boat. Let H represent the number of boat rental hours. Write and solve an equation to determine how many hours you rented the boat.

5. Solve each equation for the indicated variable.

a. Solve for $w$: $P = 2l + 2w$

b. Solve for $b$: $A = \frac{1}{2}bh$

c. Solve for $y$: $3x - 2y = 4$

d. Solve for $x$: $3x - 2y = 4$

e. Solve for $h$: $V = lwh$

f. Solve for $w$: $V = lwh$

6. Work each of the following. Round to the nearest hundredth as needed.

a. The literal equation, $D = rt$, indicates that distance traveled, D, is computed as rate of travel multiplied by time traveled. Use this equation to determine how long it takes a person to travel 675 miles if their speed averages 55 miles per hour.

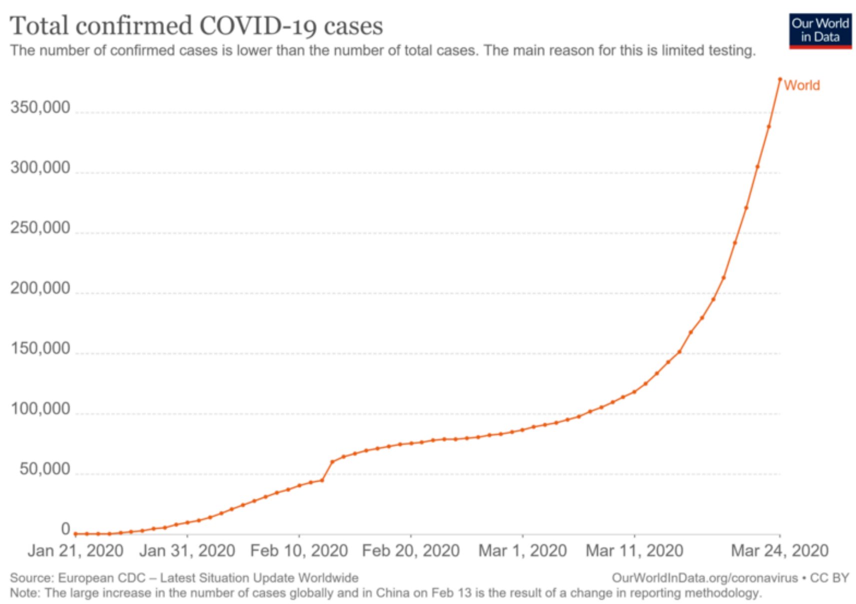
b. The literal equation, $D = rt$, indicates that distance traveled, D, is computed as rate of travel multiplied by time traveled. Use this equation to determine how far you travel if your average rate of speed is 42 miles an hour and you travel for 6 hours.

c. The formula, $I = Prt$, gives the simple interest earned on an amount invested ($P$) at an interest rate $r$ (in decimal form) after $t$ years. If you invested $5000 for 3 years and earned $345 in interest, determine the interest rate on your investment.

d. The formula, $I = Prt$, gives the simple interest earned on an amount invested ($P$) at an interest rate $r$ (in decimal form) after $t$ years. If you invested $6000 at a rate of 3.45% and earned $567 in interest, how long was your money invested?

# LESSON 6 – DATA & GRAPHS

*The graph below shows the total confirmed COVID-19 cases in the world from January 21, 2020 through March 24, 2020. What was the growth of the virus from Jan 21 through Feb 10 compared to the growth from March 11 – March 24? *[Answer: Jan 21 – Feb 10 saw a growth of about 45,000 cases worldwide whereas March 11 – March 24 saw a growth of about 250,000 cases worldwide].*




In order to answer this question, you will need to understand how to read and interpret information in graphical form. In this lesson, you will learn to read, interpret and create graphs.

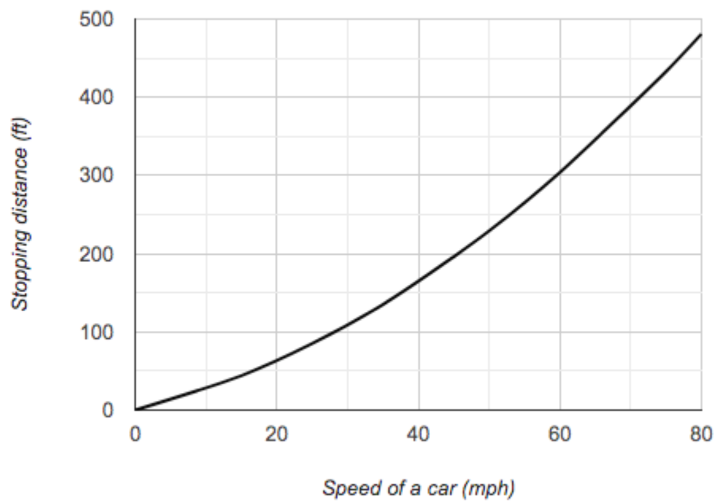


** Image source:*
https://commons.wikimedia.org/wiki/File:Total-cases-covid-19.png

## Section 6.1: Reading and Interpreting Graphs (Part 1)

Changes, trends and data in our world are represented visually using graphs. You will often see graphs, data and "infographics" in the news or online. Being able to read and interpret graphs and visual data will help you make more informed decisions in life. The examples below will help you learn more about reading and interpreting graphs.

**Video Example:**
Carefully review the given graph then use it to answer the questions below.




a. What does the horizontal axis represent (include units)?

b. What does the vertical axis represent (include units)?

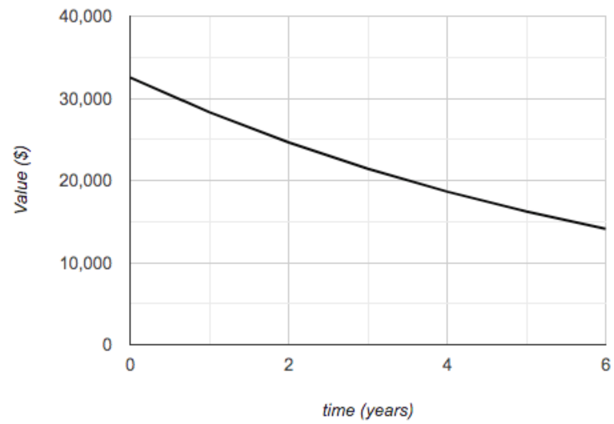
c. Is the graph increasing or decreasing?

d. Provide an overall statement of what the graph represents.

e. If vehicle speed is 60mph, about how many feet does it take to stop?

f. If 400 feet are required to stop, about how fast is the vehicle traveling?

**Video Example:**
Carefully review the given graph then use it to answer the questions below.




a. What does the horizontal axis represent (include units)?

b. What does the vertical axis represent (include units)?

c. Is the graph increasing or decreasing?

d. Provide an overall statement of what the graph represents.

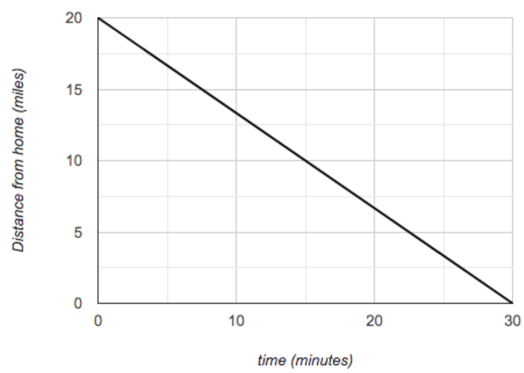
e. What is the value after 4 years, approximately?

f. After how many years does the item have value $25000?

g. What is the starting value of the item?

**You Try:**

Carefully review the given graph then use it to answer the questions below. Check your answers in Appendix A.




a. What does the horizontal axis represent (include units)?

b. What does the vertical axis represent (include units)?

c. Is the graph increasing or decreasing?

d. Provide an overall statement of what the graph represents.

e. How far is the person from home after 15 minutes?

f. When is the person 20 miles from home?

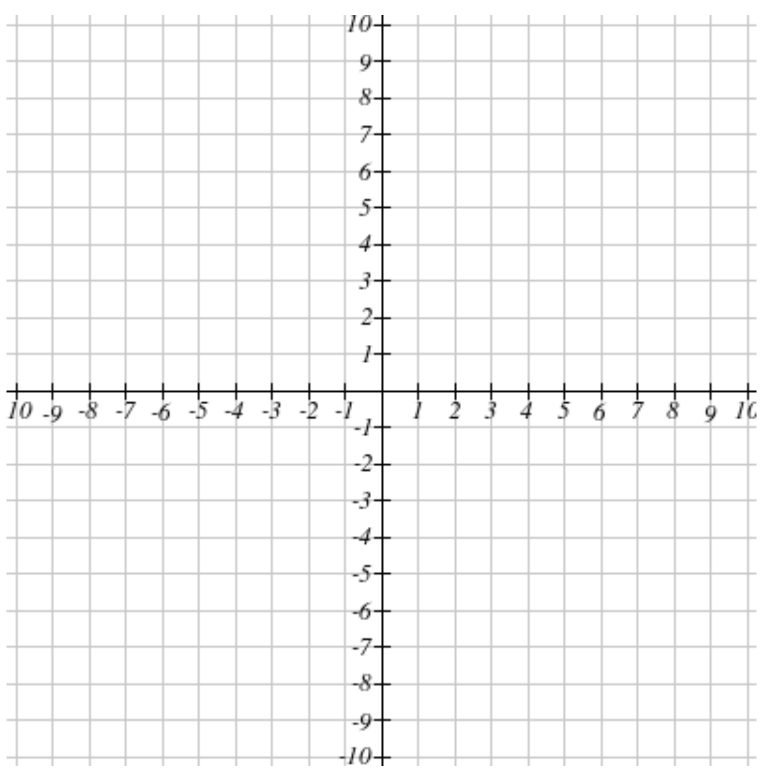
g. How long does it take for the person to get home?

## Section 6.2: The Cartesian Coordinate Plane

All three graphs in the previous section were displayed on a mathematical grid system called the Cartesian Coordinate Plane or Cartesian Coordinate System. This is a two-dimensional graphing system in which points (also called ordered pairs), contain a first coordinate and a second coordinate to fix each ordered pair in the plane. The first coordinate identifies the horizontal distance from zero and the second ordered pair identifies the vertical distance from zero. The first coordinate often is described as the "input" coordinate and the second coordinate as the "output" coordinate. The input/output designation may represent a cause/effect relationship but that is not always the case.

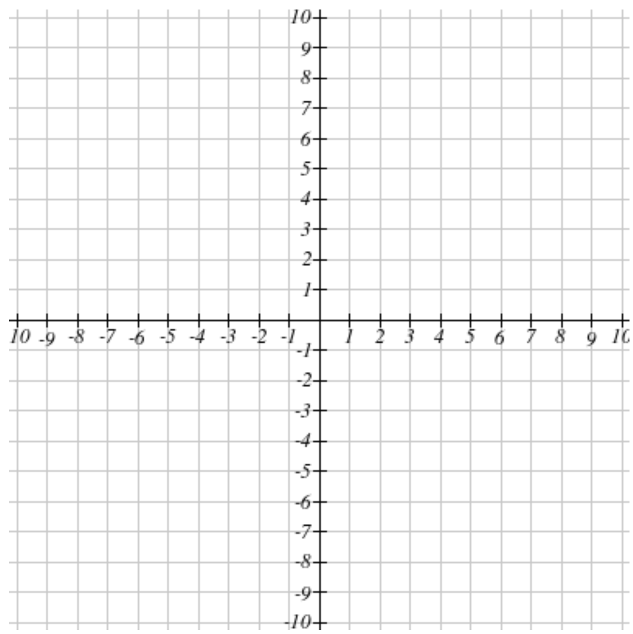
**Video Example:**
Complete the table then plot and label the ordered pairs on the coordinate system.

| Input | Output | Ordered Pairs (input, output) |
|---|---|---|
| 4 | –3 | |
| 5 | 8 | |
| | | (0, –4) |
| | | (–2, 6) |



**Video Example:**
Plot and label each of the indicated points. Name the quadrant according to the quadrant table below.

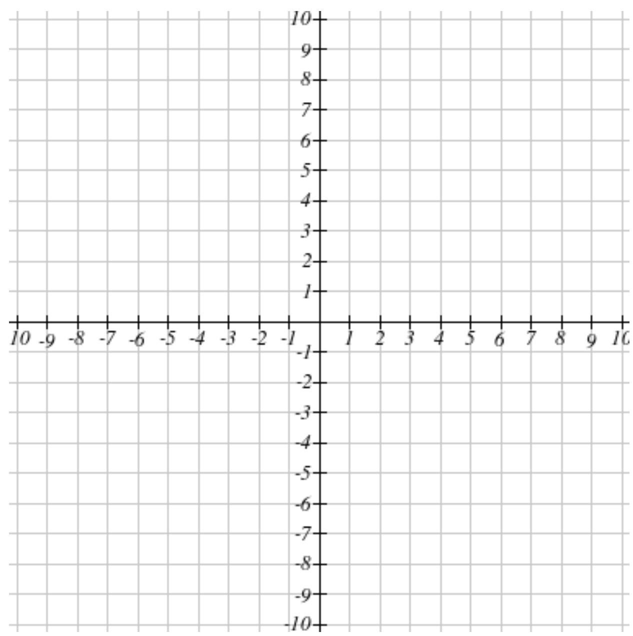
A. (–4, 2)  B. (3, 8)  C. (0, –5)  D. (–6, –4)  E. (5, 0)  F. (2, –8)  G. (0, 0)



| Quadrant | Coordinates |
|---|---|
| I | (+, +) |
| II | (–, +) |
| III | (–, –) |
| IV | (+, –) |

**You Try:**
Plot and label each of the indicated points. Name the quadrant according to the quadrant table below. Check your answers in Appendix A.

A. (6, -3)  B. (1, 9)  C. (-4, 0)  D. (–2, –8)  E. (0, 5)  F. (-9, 7)  G. (0, 0)

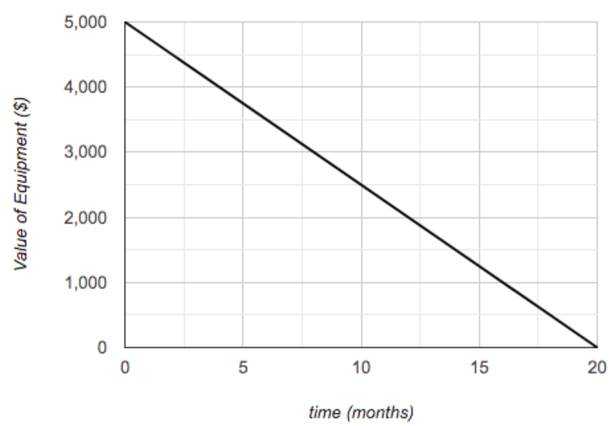


| Quadrant | Coordinates |
|---|---|
| I | (+, +) |
| II | (–, +) |
| III | (–, –) |
| IV | (+, –) |

## Section 6.3: Characteristics of Graphs

Graphs can be understood and described using the concepts of vertical and horizontal intercepts and increasing and decreasing behavior. The examples below explain these ideas in more detail.

**Video Example:**
Carefully review the graph then use it to answer the questions below.




a. What does the horizontal axis represent (include units)?

b. What does the vertical axis represent (include units)?

c. Is the graph increasing or decreasing?

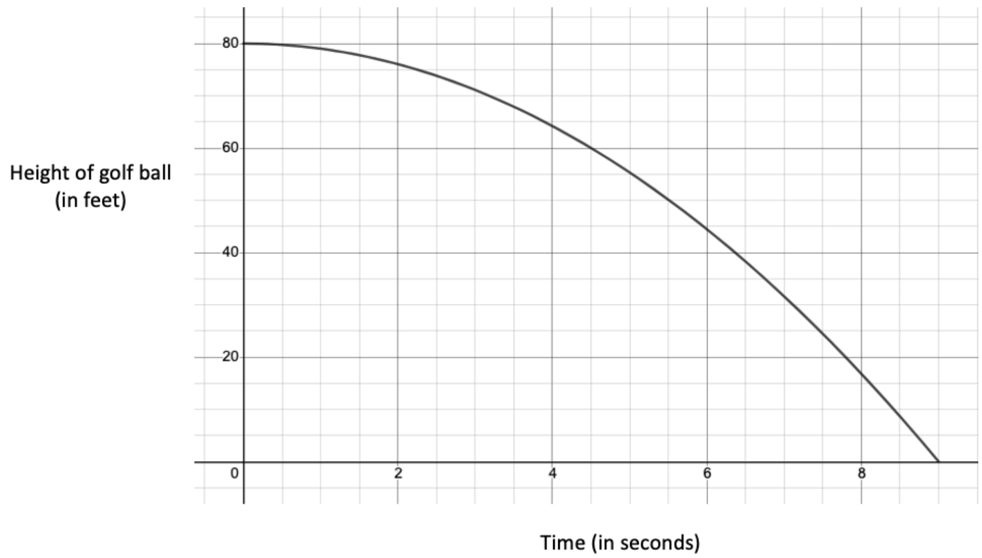
d. Provide an overall statement of what the graph represents.

e. Identify and interpret the horizontal intercept.

f. Identify and interpret the vertical intercept.

**You Try:**

Carefully review the graph then use it to answer the questions below. Check your answers in Appendix A.




a. What does the horizontal axis represent (include units)?

b. What does the vertical axis represent (include units)?

c. Is the graph increasing or decreasing?

d. Provide an overall statement of what the graph represents.

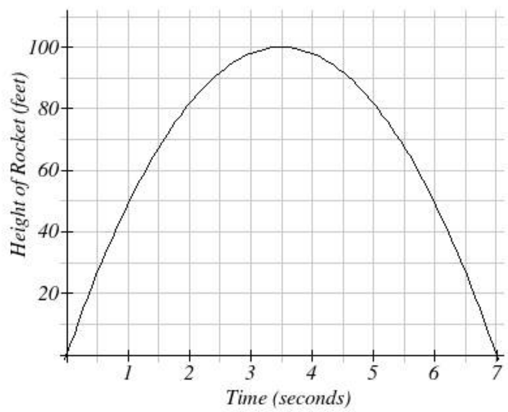
e. Identify and interpret the horizontal intercept.

f. Identify and interpret the vertical intercept.

## Section 6.4: Reading and Interpreting Graphs (Part 2)

The examples below explain how to put together all the concepts we have learned in this lesson so far and how to use those concepts to interpret the situation described by a given graph.

**Video Example:**
Carefully review the graph then use it to answer the questions below.




a. What is the input quantity (include units)?

b. What is the output quantity (include units)?

c. Summarize a situation that the graph could describe.

d. After 3.5 seconds, what is the height of the rocket?

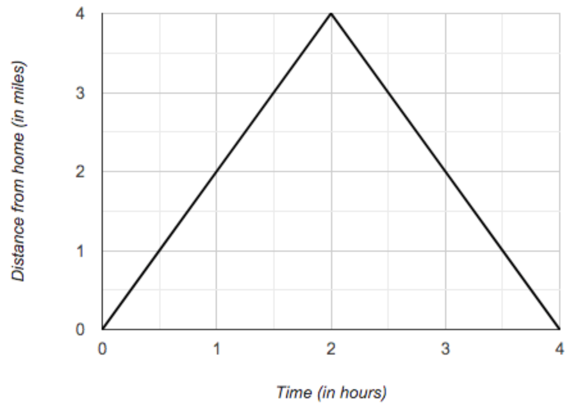
e. At what time or times is the rocket 90 feet high?

f. Identify and interpret the vertical intercept.

g. Identify and interpret the horizontal intercept(s).

**You Try:**

Carefully review the graph then use it to answer the questions below. Check your answers in Appendix A.




a. What is the input quantity (include units)?

b. What is the output quantity (include units)?

c. Summarize a situation that the graph could describe.

d. After 2 hours, what is the distance from home?

e. At what time or times is the distance from home 3 miles?

f. Identify and interpret the vertical intercept.

g. Identify and interpret the horizontal intercept(s).

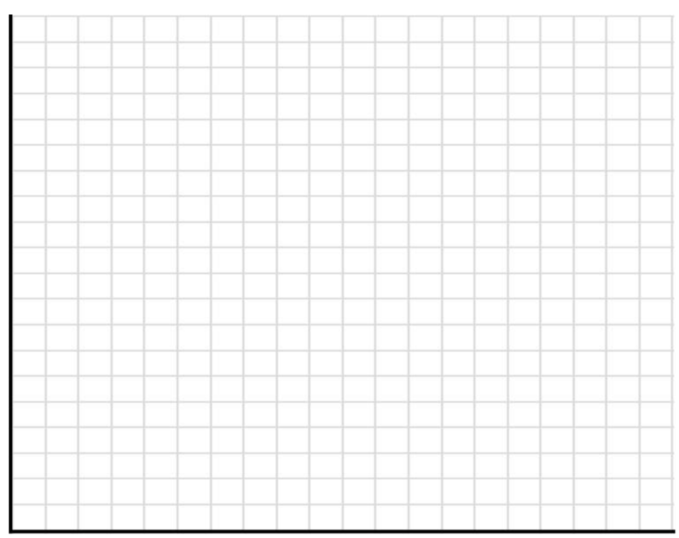
## Section 6.5: Constructing Good Graphs from Data

Up to this point in the lesson, graphs have been provided for you and you have been asked to interpret them. However, learning how to construct your own graphs is an important skill to master. Below are the criteria for creating good graphs from data sets.

1. Label your axes to fit the problem you are working on and be sure to include units.
2. Use an appropriate scale making sure the tick marks are equal distance apart on each axis.
3. The axes must meet at (0,0). Use a "//" between the origin and the first tick mark if the scale does not begin at 0.
4. Plot all points as indicated in the problem and determine if the points should be connected (continuous) or left as individual points (discrete).
5. Make full use of the available space. Don't "bunch up" your graph on one side.

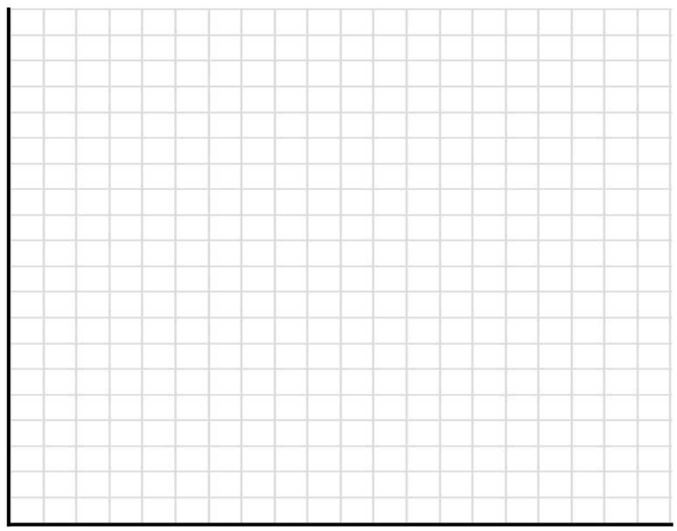
**Video Example:**
The table below shows the total distance (including reaction time and deceleration time) it takes a car traveling at various speeds to come to a complete stop. Use the criteria above to draw a good graph on the grid below.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Speed (miles per hour)** | 15 | 25 | 35 | 45 | 50 | 60 | 75 | 80 |
| **Stopping Distance (feet)** | 44 | 85 | 135 | 196 | 229 | 304 | 433 | 481 |



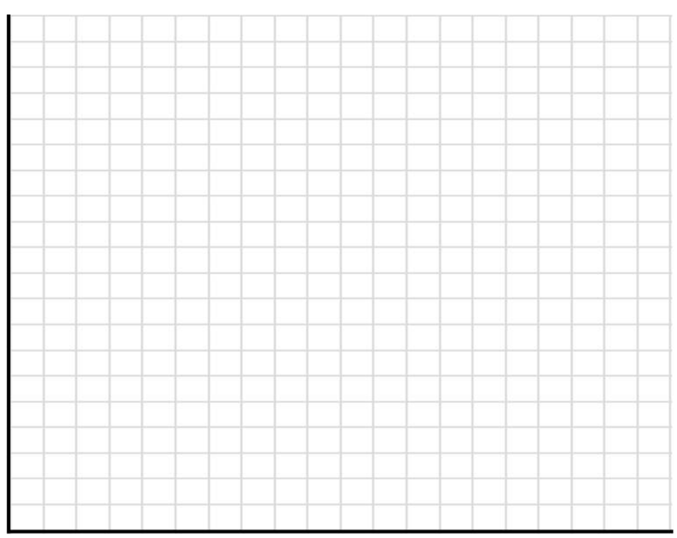
**Video Example:**
The table below shows the total cost associated with purchasing tickets for a concert. Draw a good graph on the grid below.

| **Number of tickets** | 5 | 8 | 10 | 12 | 14 | 15 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|
| **Total Cost ($)** | 625 | 1000 | 1210 | 1452 | 1694 | 1800 | 2160 | 2300 |



**You Try:**
The table below shows the height of a golf ball over time. Draw a good graph on the grid below. Check your answers in Appendix A.

| **Time (seconds)** | 0 | 1 | 1.5 | 2.4 | 3 | 3.8 | 4.5 | 5.1 |
|---|---|---|---|---|---|---|---|---|
| **Height of the ball (feet)** | 0 | 59 | 77 | 88 | 81 | 54 | 26 | 4 |



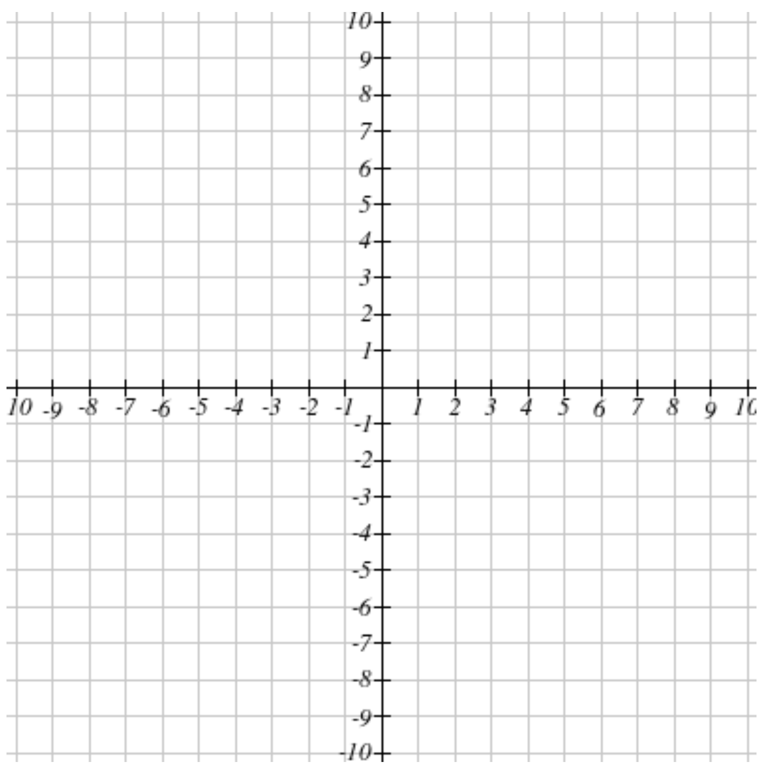
## Section 6.6: Constructing Good Graphs from Equations

Often, we are provided with an equation which defines a relationship between input and output that can be used to generate sample points and draw a graph. The criteria for drawing good graphs from equations are the same as those for drawing good graphs from data. These criteria are repeated below.

1. Label your axes to fit the problem you are working on and be sure to include units.
2. Use an appropriate scale making sure the tick marks are equal distance apart on each axis.
3. The axes must meet at (0,0). Use a "//" between the origin and the first tick mark if the scale does not begin at 0.
4. Plot all points as indicated in the problem and determine if the points should be connected (continuous) or left as individual points (discrete).
5. Make full use of the available space. Don't "bunch up" your graph on one side.

**Video Example:**
Use the equation $y = 2x - 4$ to complete the input/output table and draw a good graph of the equation on the grid below.

| **X (input)** | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|
| **Y (output)** | | | | | | | | |



**Video Example:**

Use the equation $y = 2^x$ to complete the input/output table and draw a good graph of the equation on the grid below.

| **X (input)** | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| **Y (output)** | | | | | | | |

**You Try:**

Use the equation $y = -x + 4$ to complete the input/output table and draw a good graph of the equation on the grid below. Check your answers in Appendix A.

| **X (input)** | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| **Y (output)** | | | | | | | |
| **Ordered Pair** | | | | | | | |

## Section 6.7: Using Rates of Change to Build Tables and Graphs

When we are given information about the rate at which one quantity changes with respect to another, we can use that information to create a data table and then a graph that helps us visualize the situation. The examples below illustrate this idea.

**Video Example:**

a. A local carpet cleaning company charges $15 for each room cleaned plus a nonrefundable reservation fee of $25. Use this information to create a data table and graph that describe this situation.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Input (______________)** | | | | | | | |
| **Output (____________)** | | | | | | | |



b. Water is leaking out of a tank at a constant rate of 2 gallons per minute. The tank initially held 12 gallons of water. Use this information to create a data table and graph that describe this situation.

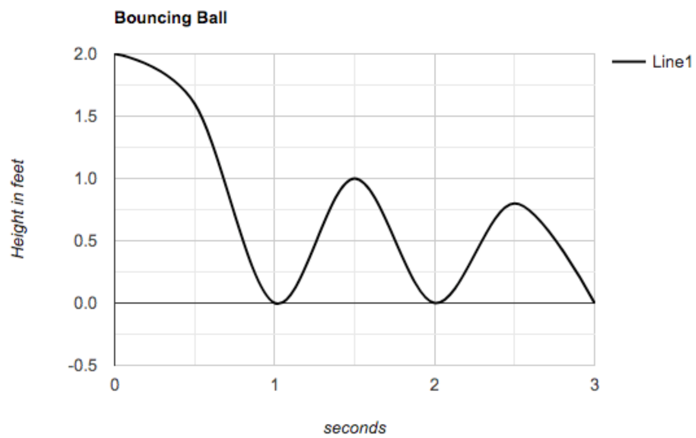
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Input (______________)** | | | | | | | |
| **Output (____________)** | | | | | | | |

**You Try:**
Sara is selling snow cones at the state fair for $3 each. Each snow cone costs $0.50 for her to make. Complete the table below to show her profit from the sale of snow cones in a given day and draw a good graph to help visualize the situation. Check your answers in Appendix A.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Input (________________)** | | | | | | | |
| **Output (______________)** | | | | | | | |

Lesson 6 – Practice & Review

1. Carefully review the given graph then use it to answer the questions below.




a. What does the horizontal axis represent (include units)?

b. What does the vertical axis represent (include units)?

c. Is the graph increasing, decreasing or both?

d. Provide an overall statement of what the graph could represent.

e. How many times does the ball hit the ground and after how many seconds?

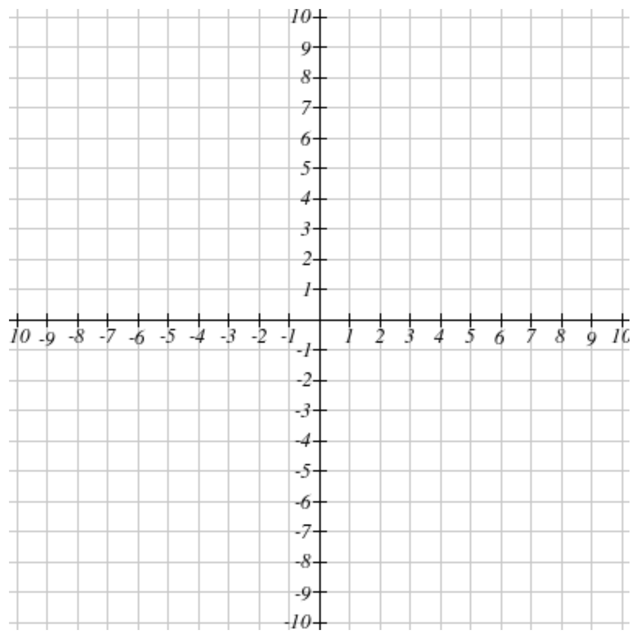
f. How high does the ball bounce at 1.5 seconds?

g. What is the maximum height of the ball?

2. Plot and label each of the indicated points. Name the quadrant according to the quadrant table below.

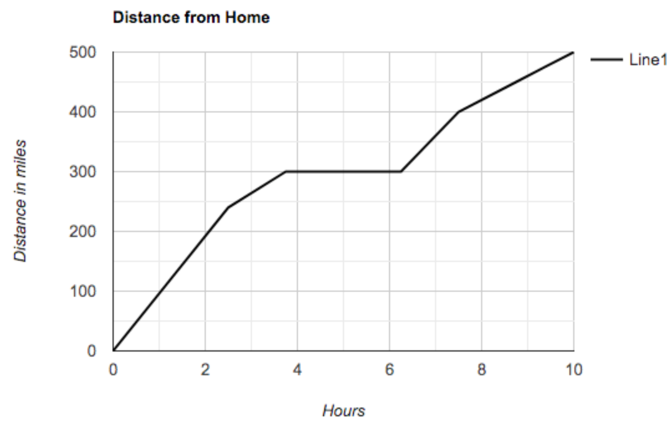
A. (0,0)  B. (-2, -1)  C. (4, 0)  D. (8, 2)  E. (-7, 0)  F. (0, -6)  G. (-6, 4)

H. (0,9)  I. (2, -4)  J. (4, -7)  K. (-3, -3)  L. (9, 7)  M. (-9, 7)  N. (-3, 8)



| Quadrant | Coordinates |
|---|---|
| I | (+, +) |
| II | (−, +) |
| III | (−, −) |
| IV | (+, −) |

3. Carefully review the graph then use it to answer the questions below.




a. What is the input quantity (include units)?

b. What is the output quantity (include units)?

c. Summarize a situation that the graph could describe. Be sure to include details about what might be happening between 4 hours and 6 hours.

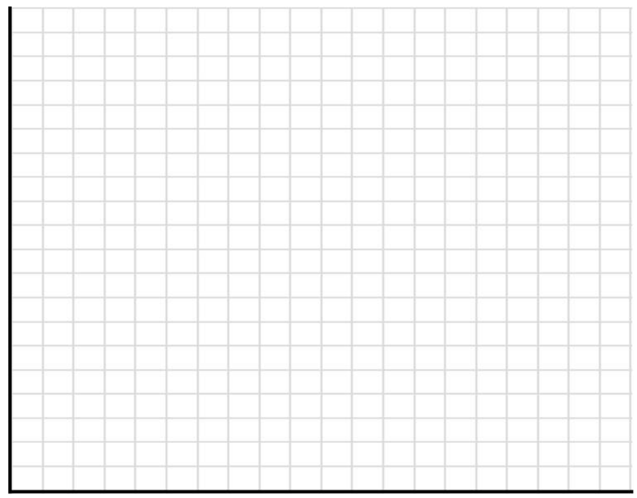
d. After 2 hours, how far from home are you?

e. Estimate the number of hours at which you are 400 miles from home.

f. How long did it take for you to reach 500 miles from home?

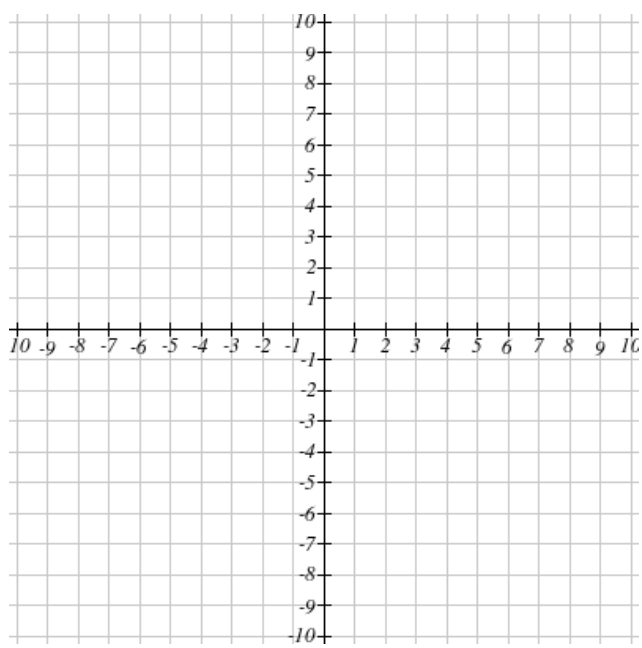
4. Sally was a contestant on the show Alone in which participants live by themselves in the woods for as long as they can. She was weighed every few weeks and her weight recorded as indicated below. Draw a good graph of these data on the grid below.

| **Time (in weeks)** | 0 | 1 | 4 | 7 | 10 | 12 | 15 | 18 |
|---|---|---|---|---|---|---|---|---|
| **Weight (in pounds)** | 185 | 180 | 172 | 160 | 145 | 140 | 130 | 120 |



5. Use the equation $y = -x + 1$ to complete the input/output table and draw a good graph of the equation on the grid below.

| **X (input)** | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| **Y (output)** | | | | | | | | |



6. Use the equation $y = (1.85)^x$ to complete the input/output table and draw a good graph of the equation on the grid below. Round values to two decimal places as needed.

| **X (input)** | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| **Y (output)** | | | | | | | |

7. Anton joined Weight Watchers and was hoping to lose 60 pounds in 6 months or 10 pounds a month. His starting weight was 285 pounds. Create a data table with data points for the first 6 months to show what his weight should be each month if he meets his goal. Create a good graph from the data table.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Input (_______________)** | | | | | | | |
| **Output (_____________)** | | | | | | | |

# APPENDIX A – YOU TRY ANSWERS

## Lesson 1

### Section 1.1

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 0 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 0 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 0 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

### Section 1.2

a. 1 b. 144 c. 823,543 d. 8

### Section 1.3

a. 15 b. 7 c. 6 d. 12

### Section 1.4

a. -59, +324, -235.34, +235.54

b. >, =, =, <

c. The Chaotic Coaster drops the most by 11 feet.

### Section 1.5

a. -5 b. 0 c. -13 d. -14
e. -4 f. 5 g. -5 h. 21
i. 0 j. 4 k. 4 l. 36
m. -16 n. 0 o. -4 p. -3

**Section 1.6**

| | | | |
|---|---|---|---|
| a. 4 | b. -4 | c. -8 | d. -8 |
| e. -4 | f. 8 | g. 16 | h. -32 |

**Section 1.7**

| | | | |
|---|---|---|---|
| a. -14 | b. -6 | c. 7 | d. 16 |

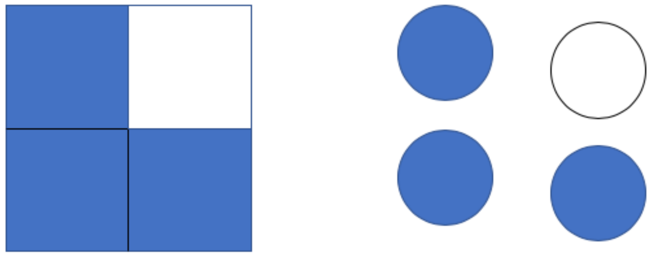
**Section 1.8**

a. Your dues for the first year are $633.
b. Your total after the five rolls is 350 points.
c. Alice should buy 240 pounds of dog food.

Lesson 2

**Section 2.1**

Answers will vary.



a. $3\frac{1}{2}=\frac{2\bullet 3+1}{2}=\frac{6+1}{2}=\frac{7}{2}$

b i. $\frac{10}{3}$ ii. $\frac{58}{7}$ iii. $\frac{22}{5}$ iv. $\frac{3}{1}$

c. $57\div 11=5R2$ $\frac{57}{11}=5\frac{2}{11}$

d. i. $4\frac{1}{4}$ ii. $7\frac{1}{6}$ iii. $4\frac{0}{9}=4$ iv. $8\frac{0}{1}=8$

**Section 2.2**

a. i. $-\frac{1}{3}$ ii. $\frac{16}{27}$ iii. $\frac{5}{3}$ iv. $-4$

b. i. 0 ii. 1 iii. Not Defined

**Section 2.3**

a. Jorge's family visited $\frac{19}{25}$ of the states in America.

b. $\frac{1}{4}$ of the marbles were Comet marbles. $\frac{3}{4}$ of the marbles were NOT Comet marbles.

**Section 2.4**

a. $\frac{7}{3}$ b. $2$ c. $\frac{17}{4}$ d. $\frac{1}{5}$

e. $\frac{6}{13}$ f. $-\frac{1}{4}$ g. $\frac{3}{2}$ h. $\frac{2}{3}$

**Section 2.5**

a. $\frac{57}{40}$ b. $\frac{66}{35}$ c. $\frac{91}{45}$ d. $\frac{1}{18}$

e. $\frac{1}{15}$ f. $-\frac{1}{15}$ g. $-\frac{7}{4}$ h. $\frac{23}{30}$

**Section 2.6**

a. Kayla and Sara ate $\frac{7}{12}$ of the pie.

b. $\frac{5}{12}$ of the pie was left over.

c. Sara ate $\frac{1}{12}$ of the pie more than Kayla.

**Section 2.7**

a. $\frac{3}{5}$ b. $\frac{21}{11}$ c. $15$ d. $\frac{1}{12}$

**Section 2.8**

1175 songs are in the Hip Hop/Rap category.

**Section 2.9**

a. $\frac{1}{15}$ b. $\frac{7}{4}$ c. $\frac{13}{2}$ d. $3$

**Section 2.10**

a. Sally could cut 80 sections of length $1\frac{1}{4}$ feet.

b. Sally would have 320 logs in total.

**Section 2.11**

a. $-\frac{5}{4}$ b. $-\frac{21}{40}$

## Lesson 3

**Section 3.1**

a. Twelve and six-hundred seventeen thousandths; 12.62; 6 is in the tenths place

b. Three-thousand four-hundred sixty-five and five hundredths; 3470; 4 is in the hundreds place

c. Five-hundred twenty-five and four-hundred sixty-seven thousandths; 500; 7 is in the thousandths place

d. Twelve-thousand four-hundred fifty-six and ninety-nine hundredths; 12457; first 9 is in the tenths place

**Section 3.2**

a. -15.532 b. 2.219 c. 38.377

d. 1560.481 e. 341.101 f. 43532.268

**Section 3.3**

a. i. 3,820,000 ii. 0.000503

b. i. $1.6 \times 10^{-7}$ ii. $5.34 \times 10^{6}$

c. i. $6.05 \times 10^{-4}$; 0.000605 ii. $1.07 \times 10^{5}$; 107,000

**Section 3.4**

a. $20.05 b. $312.00

**Section 3.5**

a. Estimate: 22; Actual: 22.034 b. Estimate: 5; Actual 5.643

**Section 3.6**

Her discount was $0.20 per gallon for the month of October.

**Section 3.7**

a. i. Five and three-hundred seventy-five thousandths; $\frac{43}{8}$

ii. Twenty-five thousandths; $\frac{1}{40}$ iii. Six-hundred twenty-five thousandths; $\frac{5}{8}$

b. i. 0.67 ii. 1.25 iii. 0.90

**Section 3.8**

a. $3.05, 3.055, 3.5, 3.55, 3.555, 3\frac{5}{9}$ b. $-1.27, -1.26, -1.255, -1\frac{1}{4}, -1.205$

**Section 3.9**

a. 75% of the students in the film program had already made their own short films.

b. $\frac{5}{25} = \frac{1}{5}$ =0.20=20% of the pennies were made prior to 1980.

c.

| Fraction | Decimal | Percent |
|---|---|---|
| $\frac{1}{8}$ | 0.125 | 12.5% |
| $\frac{1}{16}$ | O.0625 | 6.25% |
| $\frac{4}{5}$ | 0.80 | 80% |
| $\frac{5}{4}$ | 1.25% | 125% |

**Section 3.10**

a. 48.60 b. 525.48 c. 45.70 d. 50%

**Section 3.11**

a. 50.2% of the electoral votes are required to win the presidential election in the U.S.

b. The total cost of the vehicle, including tax, is $5792.90.

c. You will pay $210 for the violin after the discount.

Lesson 4

**Section 4.1**

a. $\frac{5}{10}=\frac{1}{2}$; Not a rate

b. $\frac{12\text{geese}}{15\text{ducks}}=\frac{4\text{geese}}{5\text{ducks}}$; Rate

**Section 4.2**

The unit rate for Option 1 is: 4.29 cents per pill or \$0.0429 per pill
The unit rate for Option 2 is: 4.32 cents per pill or \$0.0433 per pill
Option 1 is the better buy. [Note: Need to round to 4 decimal places to see the difference in unit rates.]

**Section 4.3**

a. x = 6

b. p = $\frac{50}{6}\approx 8.33$

**Section 4.4**

It will take 7 hours to travel a distance of 3.5 miles.

**Section 4.5**

a. $80n$

b. $450 + 38h$

c. $3.50P + 1.50S$

**Section 4.6**

a. -39

b. -1

c. The cost for 5 slices of pizza and 3 sodas is \$22.

d. The sales tax on an item that costs \$101.21 is \$9.92.

**Section 4.7**

a. i. $2x^5;2$
$-3x^3;-3$
$-2x^5;-2$
$7;7$

ii. $2b;2$
$-5b;-5$
$7b;7$
$-b;-1$

iii. $\frac{2y}{3};\frac{2}{3}$
$3y^2;3$
$-3y;-3$
$5;5$

b. i. $-3x^3+7$

ii. $3b$

iii. $-\frac{7y}{3}+3y^2+5$

**Section 4.8**

a. $x-2$ b. $2w+1$ c. $2x-6$

**Section 4.9**

a. Paul made \$2640 profit selling 2000 bottles of water.

b. The height of the ball after 3 seconds is 96 ft.

Lesson 5

**Section 5.1**

a. 2((-1) + 2) = 2(1) = 2 which checks

b. No, t = 4 is not a solution to the given equation.

**Section 5.2**

| | | | |
|---|---|---|---|
| a. $x = 2$ | b. $w = 9$ | c. $v = 19.5$ | d. $x = 3$ |

**Section 5.3**

| | | | |
|---|---|---|---|
| a. $x = -3$ | b. $t = 2$ | c. $w = -16$ | d. $n \approx -3.33$ |

**Section 5.4**

| | | | |
|---|---|---|---|
| a. $x = 18$ | b. $w = 8$ | c. $x = -12.1$ | d. $w = 1$ |

**Section 5.5**

| | | | |
|---|---|---|---|
| a. $x = -12$ | b. $w = 1.5$ | c. $m = -1$ | d. $x \approx 1.96$ |

**Section 5.6**

a. If you have $800 to spend on tuition, you can take a maximum of 10 credits.

b. Paul needs to sell 40 bottles of water to break even.

**Section 5.7**

a. The landscapers worked for 4.5 hours or 4 hours and 30 minutes.

b. You would have to talk 68 minutes each month for the two plans to cost the same amount.

**Section 5.8**

a. $C = D - A - B$

b. $b = \dfrac{8}{ac}$

c. $y = 5x - 2$

d. $x = \dfrac{y+2}{5}$

e. $\dfrac{9}{5}C + 32 = F$

f. $\dfrac{A-P}{Pt} = r$

**Section 5.9**

a. The rate of travel is 73 miles per hour (mph).

b. The length of the given rectangle is 10 inches.

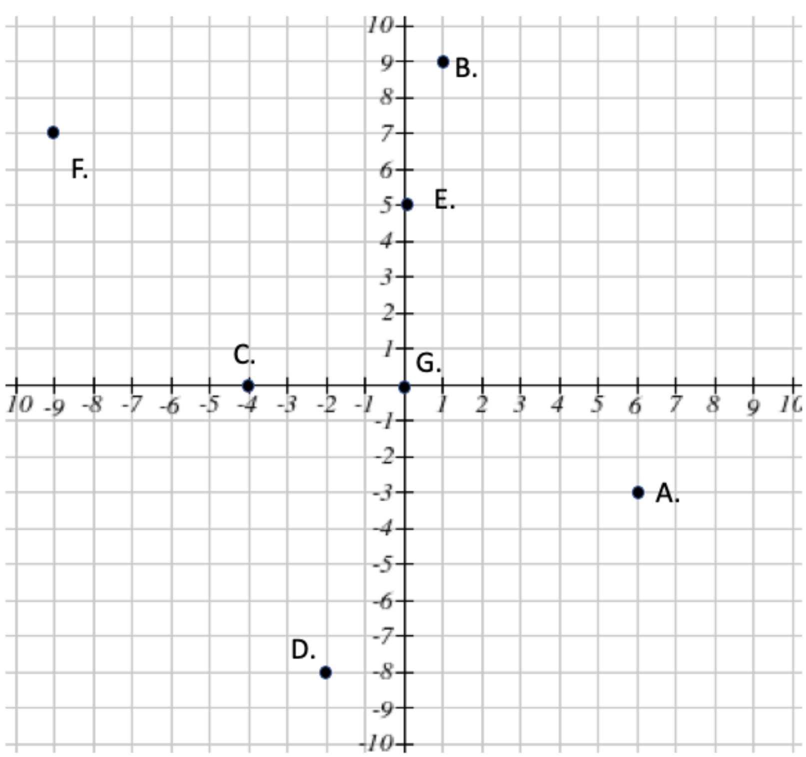
Lesson 6

**Section 6.1**

a. Time in minutes
b. Distance from home in miles
c. Decreasing
d. The graph shows the distance from home (in miles) depending on the time elapsed (in minutes)
e. 10 miles
f. When t = 0 (beginning of the trip going home)
g. 30 minutes

**Section 6.2**

a. (6, -3) – Q IV
b. (1, 9) – Q I
c. (-4, 0) – None – on horizontal axis
d. (-2, -8) – Q III
e. (0, 5) – None – on vertical axis
f. (-9, 7) – Q II
g. (0,0) – None (origin)



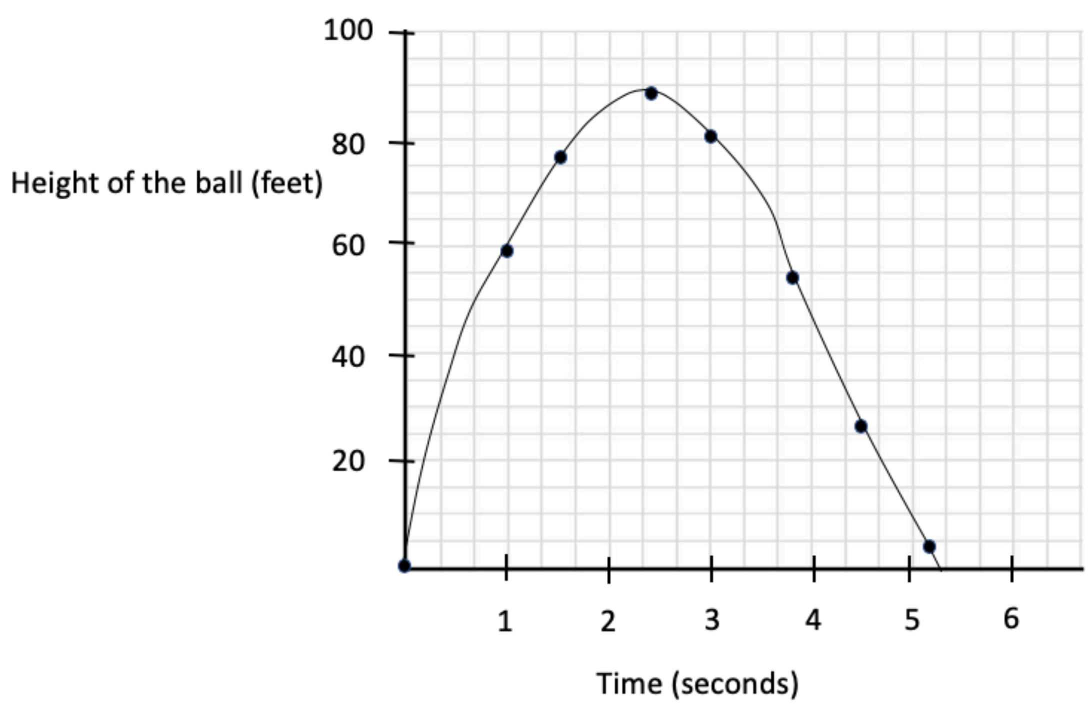
**Section 6.3**

a. Time in seconds
b. Height of a golf ball in feet
c. Decreasing
d. The graph represents the height of a golf ball (in feet) over time elapsed (in seconds)
e. (9, 0) The golf ball hit the ground after 9 seconds.
f. (0, 80). A golf ball started at a height of 80 feet.

**Section 6.4**

a. Time in hours
b. Distance from home in miles
c. A person starts a trip leaving from home, reaches a destination that is 4 miles from home, then returns home.
d. 4 miles
e. At 1.5 hours and 2.5 hours
f. (0, 0) The person starts the trip (time t = 0) at home (distance from home = 0 miles)
g. (0, 0) The person starts the trip (time t = 0) at home (distance from home = 0 miles)
(4, 0) The person returns home from the trip after 4 hours (distance from home = 0 miles)

**Section 6.5**

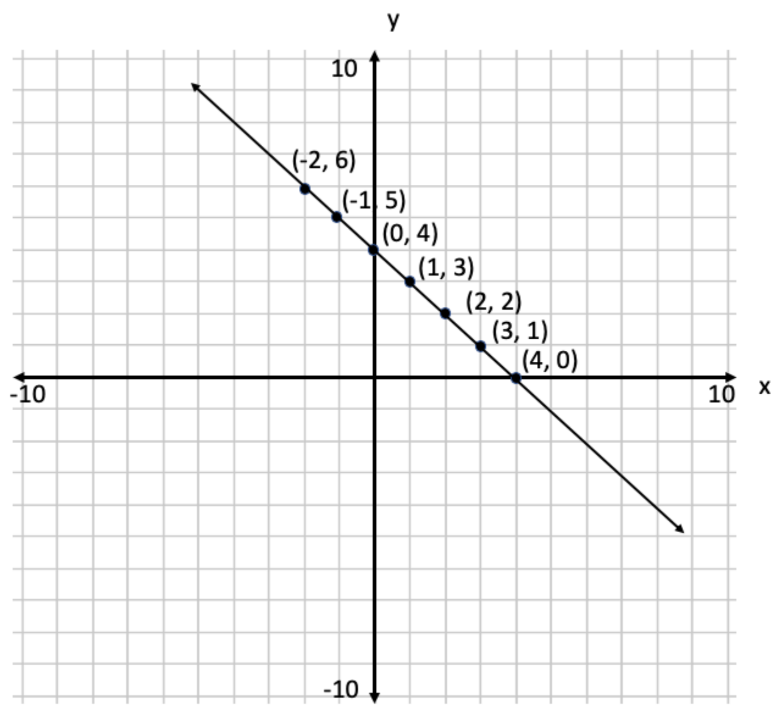


**Section 6.6**

| X (input) | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| Y (output) | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

Ordered Pairs: (-2, 6); (-1,5); (0, 4); (1, 3); (2, 2); (3, 1); (4, 0)

Note that these points are connected as we are working with an equation and can use decimal values to



**Section 6.7**

To compute profit, use the equation P = 2.50C where C is the number of cones sold in a day.

| Input: Snow cones sold per day | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Output: Profit in $ | 0 | 2.5 | 5 | 7.5 | 10 | 12.5 | 15 |

Note that these points are not connected as we cannot sell decimal numbers of snow cones.